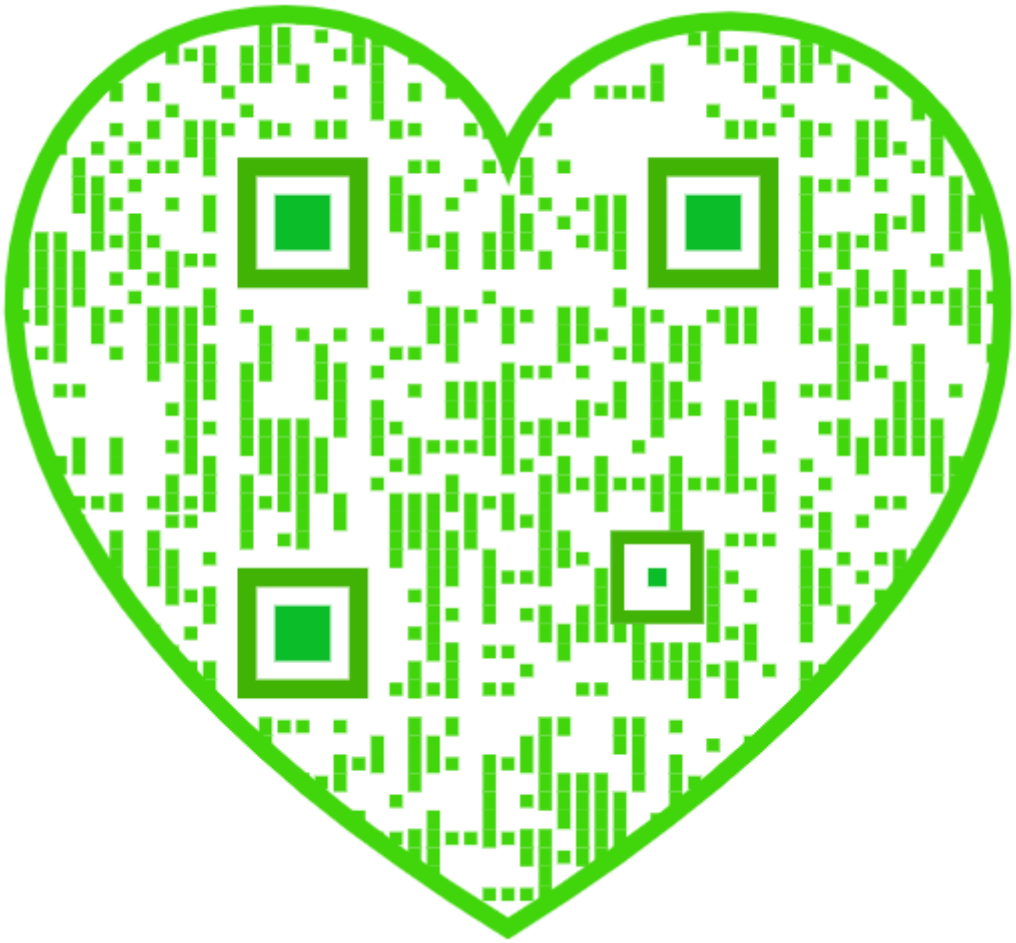

The "Westminster" Series

NATURAL SOURCES OF POWER

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BY

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PREFACE

Two departments of engineering and their applications to industry form the subjects of the following pages. While the subject of water power (which occupies the first and greater half of the volume) would be conceded to be more important than that of wind power, the latter has a wider application at the present time than is popularly assigned to it, for the demand for windmills was never so great as it is to-day, nor the trade of the manufacturer of such motors never so brisk, and there are unmistakable signs of abnormal expansion in the direction of their useful application in the great agricultural countries of the world. The author cannot be charged with any attempt to appraise the relative value to the industrial world of two natural sources of power so widely different in distribution and reliability, moreover in a given place the choice rarely lies between the two, as local conditions determine the preference should an alternative perchance exist. It may, however, be said that the use of power of either kind in small installations renders such comparison more likely to be effective, for at present large windmill installations do not exist, with which water-power plants of great concentrated power might be compared.

The principles which underlie this utilisation of natural sources of power are those comprised in a few elementary laws of mechanics, but the complex problems which the

engineer has to face and which demand special and extensive treatment require extended space, and moreover they cannot be satisfactorily elucidated without recourse to the calculus. With therefore an imposed limit to the use of mathematics, the object in view has been mainly that of showing that if the available power be small it may nevertheless be well worth turning to account, and if the pages that follow confer upon the reader a sense of the potentialities of the smaller bounties of nature they will have discharged their office. The thanks of the author are due to many persons and firms for assistance rendered. Besides other acknowledgments made throughout the text he wishes to express his indebtedness to Messrs. Holman Bros. of Canterbury, who generously placed valuable drawings at his disposal, to the Council of the Royal Agricultural Society for permission to reproduce illustrations of wind engines, to Mr. W. Halliwell for kind assistance, and to the proprietors of "Engineering" for permission to reproduce illustrations contained in a paper which he presented to the British Association, and for other articles contributed to their columns. He also gratefully acknowledges valuable advice from Mr. Talbot Peel, M.A., and Mr. Alph Steiger, M.Inst.C.E.

Among the illustrations are many supplied by the following firms and engineers: Gilbert Gilkes & Co. Ltd., W. Gunther & Sons, Garrick & Ritchie, Theodor Bell & Co., Escher, Wyss & Co., Rickman & Co., Thomas & Son, and J. W. Titt, to all of whom the author extends his thanks.

R. S. B.

WESTMINSTER,
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UNITS WITH METRIC EQUIVALENTS AND ABBREVIATIONS

LENGTH AND DISTANCE.

Unit.	Abbreviation.
1 inch	in.
= 25·4 millimetres.	
= 2·54 centimetres.	
= 0·0254 metres.	
1 foot	ft.
= 304·8 millimetres.	
= 30·48 centimetres.	
= 0·3048 metres.	
1 yard	yd.
= 3 feet.	
= 0·914 metres.	
1 mile	mile.
= 5,280 feet.	
= 1,609 metres.	
= 1·609 kilometres.	
1 millimetre	mm.
= 0·0394 inches.	
1 centimetre	cm.
= 0·394 inches.	
1 metre	m.
= 39·37 inches.	
= 3·28 feet.	
1 kilometre	km.
= 0·621 miles.	

SURFACE AND AREA.

Unit.	Abbreviation.
1 square inch	sq. in.
= 645·2 square millimetres.	
= 6·452 square centimetres.	
1 square foot	sq. ft.
= 0·111 square yards.	
= 0·0929 square metres.	
1 square yard	sq. yd.
= 0·836 square metres.	
1 square millimetre	sq. mm.
= 0·00155 square inches.	
1 square centimetre	sq. cm.
= 0·155 square inches.	
1 square metre	sq. m.
= 10·76 square feet.	

VOLUMES.

1 cubic foot	cu. ft.
= 0·0283 cubic metres.	
= 28·32 litres.	
1 cubic metre	cu. m.
= 1·308 cubic yards.	
= 35·31 cubic feet.	

WEIGHTS OR MEASURES.

1 pound	lb.
= 0·454 kilogrammes.	
1 ton (English)	ton.
= 2,240 lbs.	
= 1·016 metric tons.	
*1 ton (American)	
= 2,000 lbs.	
= 0·907 metric tons.	
1 kilogramme	kg.
= 2·205 lbs.	

* Unless otherwise stated, the "long" ton of 2,240 lbs. is used throughout this volume. It is about 2 per cent. greater than the metric ton.

Unit.	Abbreviation.
1 ton (metric)	ton (metric).
= 1,000 kilogrammes.	
= 0·984 tons (English).	
= 1·102 tons (American).	

PRESSURES.

1 pound per square inch	lb. per sq. in.
= 0·0703 kilogrammes per square centimetre	
= 2·307 feet water column.	
= 2·036 inches mercury column.	
1 kilogramme per square centimetre . .	kg. per sq. cm.
= 14·22 pounds per square inch.	
= 0·968 atmospheres.	
= 10 metres water column.	
1 atmosphere*	at.
= 29·92 inches mercury column.	
= 760 millimetres mercury column.	
= 33·9 feet water column.	
= 14·7 pounds per square inch.	
= 2,116 pounds per square foot.	
= 1·033 kilogrammes per square centimetre.	

LINEAR VELOCITIES.

1 foot per second	ft. per sec.
= 0·682 miles per hour.	
= 0·305 metres per second.	
= 1·097 kilometres per hour.	
1 mile per hour.	ml. per hr.
= 1·467 feet per second.	
= 88 feet per minute.	
= 0·447 metres per second.	
= 1·609 kilometres per hour.	
1 metre per second	m. per sec.
= 3·28 feet per second.	
= 2·24 miles per hour.	
= 3·6 kilometres per hour.	
1 kilometre per hour	km. per hr.
= 0·911 feet per second.	
= 0·621 miles per hour.	

* Barometric variation necessarily renders an "atmosphere" a purely arbitrary quantity. It is generally taken as above.

ANGULAR VELOCITIES.

Unit.	Abbreviation.
1 revolution per second	rev. per sec.
= 6·28 radians per second.	

ACCELERATION.

1 foot per second per second	ft. per sec. per sec.
= 0·682 miles per hour per second.	
= 1·097 kilometres per hour per second.	

ENERGY.

1 foot-pound	ft.-lb.
= 0·138 kilogramme-metres.	
1 kilogramme-metre	kg.-m.
= 7·233 foot-pounds.	
1 British thermal unit	B.T.U.
= 778 foot-pounds.	
= 0·252 calories.	
= 107·6 kilogramme-metres.	
1 calorie	cal.
= 426·9 kilogramme-metres.	
= 3,088 foot-pounds.	
= 0·001163 kilowatt-hours.	
1 kilowatt-hour	kw. hr.
= 2,655,403 foot-pounds.	
= 367,123 kilogramme-metres.	
= 859·975 calories.	

POWER.

1 horse-power	H.P.
= 33,000 foot-pounds per minute.	
= 550 foot-pounds per second.	
= 76·04 kilogramme-metres per second.	
= 745·6 watts.	
1 horse-power (metric)	H.P. (metric.)
(force de cheval or Pferde-kraft)	
= 75 kilogramme-metres per second.	
= 0·986 horse-power.	
= 735·5 watts.	

N.B.—Physical constants and units relating to water are to be found in the table on p. 45. The weight of dry air at atmospheric pressure and at a temperature of 10° C. (50° F.) is 1,247 grammes per cubic metre, or 0·08 lbs. per cubic foot.

NATURAL SOURCES OF POWER.

CHAPTER I.

INTRODUCTORY.

THE supplies of energy upon which the engineer draws for his wants are of several kinds, and though mutually convertible, are distinct in their natural states, and bear separate characteristics. To some of them it is necessary to apply a transformation process before they are in the form in which the energy can be used for the purpose in view. In other cases transformation is not necessary, and nature yields up her stores in what may be termed a manufactured form. This distinction, arbitrary as it is, is convenient for the purpose of introducing the subject, and for defending the title under which these pages appear, which expressly classifies the sources of power considered as "natural." In a sense, therefore, the "natural" sources, such as water and wind power, are those which the engineer utilises directly for propelling machinery, and which directly supply mechanical energy without any intermediate stage of transformation; while fuels may, in contradistinction, be placed in another classification comprising

those bounties of nature which have to undergo chemical decomposition by which heat energy is developed, which is again transformed into mechanical energy by some form of engine or appliance.

Long before the birth of the great inventions, by which the engineer is enabled to use the stored sunbeams in coal, gas, and oil, and by means of which heat is transformed into mechanical energy, the natural sources were tapped to supply the comparatively small requirements of a period long antecedent to the present age. To these sources of power, inexhaustible as they are, the engineer will some day turn again when our coal measures are exhausted, our gas retorts cold and empty, and the oil wells of the earth dried up for ever. In the present age of fuel, when almost all mechanical ingenuity is directed towards the utilisation of coal and other natural fuels, when engineers make their reputations by devising machinery to accelerate the consumption of this black diamond, and at a time when our extravagance is at its height, scarcely sufficient thought is directed to these natural powers, water and wind, which will some day be in the ascendant again as they were in the past, when sailing vessels ploughed the ocean long before steam power was introduced with its accompanying profligate waste of our fuel resources. Geologists have estimated with reasonable confidence the approximate time that will elapse before the last ton of coal is shovelled into the wasteful furnace, and while the sun itself is slowly shrinking, and therefore our source of water and wind power will one day come to an end, and with it all life upon the earth, the end of our fuel supplies is, compared with the length of time of any appreciable diminution in the energy

radiated by the sun, as one day to thousands, perhaps millions, of years. The foregoing arbitrary distinction between water and wind power on the one hand, and that derived from fuel on the other will serve to explain why these pages are limited to the former, though the comprehensive title would imply something more. Nevertheless, as all forms of energy are capable of being changed into other forms, the engineer who wishes to obtain the best results has to be conversant with the changes that are likely to occur, and the losses that he may be exposed to by the transmutation of energy into other forms than that in which it is directly used in his machines.

Water and wind power are manifested in various forms. The evaporation of water from the ocean, descending in the form of rain upon high lands and plateaux, supplies the rivers from which we are able by means of water wheels to utilise this power that the sun incessantly provides. Ocean currents and tides, though but little utilised, also represent a large and continual source of power which may some day be turned to account.

Then wind power, which has been employed for ages to drive our commerce over the seas, and which has recently been tentatively used through the novel form of wave motors, has had in the past a greater share of attention than at the present time, when power from fuel is occupying the chief place in the minds of engineers, and is such an important feature in our industrial life.

The law of the conservation of energy teaches us that energy cannot be destroyed, but that it can only be changed from one form into another. The engineer however has

frequent reason to regard energy as destroyed or lost to his purpose, as for instance when a steam engine expends energy in heating bearings, or in cylinder condensation. If these sources of loss were not present, the energy so expended would be put into useful work in helping to drive the machinery. The physicist would not regard these losses in the same light as the engineer who is accustomed to speak of them as such. In every application of natural energy to the industrial arts, heavy tolls are exacted and losses sustained in the process of application, and the skill of the engineer is mainly directed towards reducing these losses, which, according to their magnitude may mean success or failure of the enterprise upon which he is engaged. His chief occupation is to contend with the imperfections of his own apparatus which allows part of the energy to drain away in the same manner that a leaky pitcher carried from the well to a distance loses part of its contents by the way. If the water it contained were valuable, special efforts would be directed towards the replacement of the leaky vessel by a sound one; this however is always impossible in engineering, and the most the engineer can do is to diminish the loss by imperfectly plugging the leaks. With power derived from fuel upon which a high price is placed, the economy of the plant is an important consideration, and it is also becoming one of the chief factors in hydraulic power, especially where water privileges are retained at a high value. As the wind is free to everyone it is commonly supposed that windmill efficiency is a matter of little moment, but such is not the case, for the power obtained from the wind is so extremely small for the size of the wheel and the cost of the installation, that

it has to be carefully safeguarded from leakage, and the extended usefulness of the wind wheel to-day for pumping water is largely due to improvements in mechanical construction which are designed to prevent waste, and which therefore utilise the small power to the fullest extent. That the windmill is still a very inefficient machine will appear evident from what follows, while the hydraulic turbine is by far the most efficient prime mover that we possess, all heat engines taking a secondary place in this respect.

In order to be able to make a comparison, both scientific and commercial, between the efficiencies of different forms of prime mover, it is convenient to introduce a few words concerning energy, and the various forms in which the engineer has to deal with it, and the way in which it is measured.

Though energy is manifested in various forms such as mechanical, heat, or electric energy, the common denominator of them all is known, and whatever may be the form in which it appears, it may readily be converted into that most familiar unit, the foot-pound (in the metric system the kilogramme-metre), by a simple sum. Now the foot-pound is often defined as the amount of energy which is expended in raising a one-pound weight through one foot, or half a pound two feet and so on, the product of the weight and height always being unity. This definition, though correct, is misleading, for the most frequent use which the engineer has for measuring energy does not involve the *lifting* of weights at all, and therefore such a definition does not cover the majority of cases in which energy is measured or dealt with. The introduction of a pound or kilogramme into the unit is often mentally inseparable from the idea

of *weight*, and this gives rise to a notion which the writer has observed, that the process of lifting weights is exclusively connected with the measurement of energy. In order to broaden the definition so as to meet the cases in which the engineer is chiefly concerned, it is necessary to say that a foot-pound is the energy expended in overcoming a force of one pound through a distance of one foot in the direction of the pressure, or in general, when a force of p lbs. is exerted through a space of s feet, $p \times s$ foot-pounds of work are expended, and if pounds and feet be replaced by kilogrammes and metres, $p \times s$ kilogramme-metres are expended.

The definition of energy is thus independent of time, and a given number of foot-pounds may be expended during any period without invalidating the definition of the quantity of energy. This is the distinction between energy and power, for the latter involves the *rate* at which energy is expended.

The relation that exists between mechanical and heat energy, or, as it is commonly called, the mechanical equivalent of heat, is known to a high degree of accuracy. The great discovery of Joule and the subsequent accurate experiments which have been made by eminent physicists show that 778 foot-pounds converted into heat would be sufficient to raise the temperature of a pound of water 1° F. which quantity of heat is established as the British Thermal Unit.[1] Thus, the energy expended in overcoming a resistance of 500 lbs. through a distance of 2·7 feet *at any speed* is 1,350 foot-pounds, or in British thermal units

$$\frac{1,350}{778} = 1{\cdot}735 \ldots . \text{ B.T.U.}$$

[1] The calorie is the amount of heat necessary to raise the temperature of one kilogramme of water 1° Centigrade.

The imagination is no guide to the relation between the thermal and mechanical units, for it is difficult to accept the statement that the heat necessary to raise a pound of water one degree F. is actually equal to 778 foot-pounds. Possibly the explanation for this may lie in the fact that mechanical energy is more often the subject of tentative measurement by which our ideas are adjusted, yet there is probably no other experimental constant in physics upon which there is less doubt than the mechanical equivalent of heat, within the small limits of experimental error.

The electrical units of energy[1] are more confusing than the mechanical, for the reason that the foot-pound does not involve any question of time, while a watt, including as it does the ampere or rate of flow of current, is essentially concerned with time, and is therefore allied to a *power* unit. For instance, 1,000 foot-pounds are not equal to 22·6 watts, and such an equation would be meaningless, but the *power* expressed by 1,000 *foot-pounds uniformly expended in one minute* is correctly expressed in electrical units by 22·6 watts; likewise 1 foot-pound = 1·36 watt-seconds, the second term of the equation being qualified by a statement as to the time during which the 1·36 watts are expended. A pressure of 1·36 volts acting through a resistance of 1·36 ohms for one second, causing a current of one ampere thereby, expends an amount of energy in one second which, in mechanical units, is measured by one foot-pound. Foot-pounds and watts are not therefore mutually convertible,

[1] The Joule, or watt-second, which was defined at the International Congress of 1893 in Chicago as equal to 10^7 C.G.S. units of work (ergs), is represented closely by the energy expended in one second by an international ampere against an international ohm. It is equal to 0·738 foot-pounds.

neither are watts and thermal units, for the latter are definitely related to the foot-pound, and do not involve time or rate of expenditure of energy in any sense.

The foregoing fundamental distinction between the foot-pound and the watt is so important, and at the same time so easily overlooked, that an explanation may be not out of place. The difficulty rests with the proper understanding of the unit of current—the ampere. It is unfortunate that we do not generally express a measure of current in any other way than by a single word which in itself hides the important element of time,[1] or that the definition is not conveniently susceptible of subdivision, further than to say that the ampere is the current which will deposit a certain weight of silver from a nitrate solution in a second. Taking the analogous case of hydraulic measurements, it is customary to express flow in "cubic feet per second," which in itself is a compound expression, clearly including the unit of time, and also the familiar unit—the cubic foot. Moreover there is no short expression for "cubic foot per second" which would take the same place in hydraulic engineering that "ampere" does in electrical. It is thus that the "ampere," and consequently the "watt," fail to make evident the important item of time, which is hidden away in these single words, there being no analogous electrical expression for "cubic foot." To express *energy* in electrical units we have therefore to use compound terms such as "watt-hour" or "kilowatt-hour."

[1] The unit of quantity of electricity, which however is seldom used, and is universally disregarded in practical engineering, is the coulomb. It is defined as "the quantity of electricity transferred by a current of one international ampere in one second," and is that quantity which will deposit 0·001118 grammes of silver in a voltameter.

Power.

It cannot be too clearly stated that *power* involves the *rate* of expenditure of energy, and that without some knowledge of the time in which a given amount of energy is expended, no estimate of power can be made. A given number of foot-pounds may, in the course of being expended, mean millions of horse-power or a millionth of a horse-power according to the time or rate at which the expenditure takes place, and the power may vary between any limits during the process of expenditure. It is easy to see then how velocity enters into all questions involving power, for velocity is the rate at which the space varies with regard to the time. The unit of power—the horse-power—of the engineer is defined as a *rate* of expenditure of energy. It is incorrect to say that one horse-power is defined by the expenditure of 33,000 foot-pounds in one minute or 550 in one second. This is only true if the rate of expenditure is uniform throughout the time. Otherwise the power may vary widely during the time considered. A steam engine, nominally assumed to develope a certain horse-power, is continually varying in power, even in a single revolution of the crank. In consequence of the "rate" of expenditure being a measure of power, it would, for example, be meaningless to say that the power expended in moving a body against a resistance of 1,800 pounds through 8 feet was 14,400 foot-pounds; this would be the *energy* but not the *power*, and yet careless expressions of this kind are common in technical literature. If this movement was effected at a uniform velocity, and the observed time of travel

over 8 feet was 4 seconds, the power in horse-power units would be—

$$\text{H.-P.} = \frac{p \times v}{550} = \frac{1{,}800 \times \frac{8}{4}}{550} = 6{\cdot}55.$$

The velocity may be constantly varying, as it usually is in practical problems, and the power therefore varies in direct proportion. Thus, though the energy expended in this example would always be the same, regardless of the speed, the power may have any value whatever depending upon the speed at any particular instant.

Energy in any form expended at a given rate involves the expenditure of power. If, for example, eight British thermal units be absorbed by the water in a boiler in 3·2 minutes, and the rate of absorption is constant, *i.e.*, 2·5 units per minute, the horse-power of which this is an equivalent would be—

$$\text{H.-P.} = \frac{2{\cdot}5 \times 778}{33{,}000} = 0{\cdot}059.$$

It has been pointed out that the watt is a measure of power in itself, for it includes the ampere, which introduces the necessary time or rate of expenditure of energy. It is therefore correct to say that one horse-power is 745·6 watts (or 0·7456 kilowatts) in electrical units. At a first glance it would appear that something has been left out, but when it is understood that the watt *involves time*, the time qualification which must be added to a statement of mechanical energy in foot-pounds to render it in power-units will be seen to have no analogy in electrical units, in other words, 33,000 foot-pounds *expended in one minute* is expressed otherwise by 745·6 watts. If it is desired to express energy in electrical units, as it is frequently necessary

to do in hydraulic engineering, the kilowatt-hour or watt-hour can be used. This indicates an amount of energy represented by the expenditure of one kilowatt continued for one hour, which might equally well be represented by that expended by 1·34 h.-p. in the same period. The kilowatt-hour is becoming an increasingly useful unit for practical purposes, and it is just as definite as a measure of *energy* as a foot-pound or kilogramme-metre, for it stands for a definite number of each of these units, and has, *pari passu*, an equivalent in heat-units.

Efficiency of Machines.

The value placed upon a prime mover by the engineer depends to a large extent upon its efficiency, *i.e.*, the ratio of the energy derived from it to that put into it in another or the same form. But it will be evident that, as some forms of energy cost more than others, efficiency alone is not the factor that determines the commercial value of a prime mover. For example, the price of coal renders the efficiency of the steam engine a matter of greater importance than that of the hydraulic turbine supplied with abundance of water obtained at little or no cost. Likewise the windmill, using a free natural source of energy, is seldom measured by the ratio of the power obtained from it to that supplied by the wind. In the case of the steam engine a slight increase in efficiency often means a considerable reduction of the fuel bill, and establishes the success of the plant, while an increase in the efficiency of a motor supplied with a working fluid which costs nothing means an increased output, which could otherwise be obtained by an increase in the size of the engine, without material additional running cost. The

efficiency of an engine may therefore in some cases be of great commercial importance, while in others it is not the determining factor in the installation of a power plant.

The measurement of the efficiency of hydraulic and wind motors is simplified by the fact that the energy supplied is in the same form as that delivered, and therefore it is the ratio of the two that directly gives the efficiency. Thus the actual work done by an hydraulic turbine in a unit of time (the power) is measured in foot-pounds, as is also the theoretical power in the falling water. The ratio of the one to the other gives the efficiency. In heat engines, on the other hand, the energy in the form of thermal energy supplied, is converted into mechanical energy delivered by the engine, and one set of units must be converted into the other before the efficiency is known. For example, a steam engine uses 1·29 lbs. of coal per h.-p. hour. The coal has a calorific value of 14,000 B.T.U. per pound, so that the value of the fuel necessary per hour for 1 h.-p. is $1{\cdot}29 \times 14{,}000 \times 778$ foot-pounds. As the energy delivered by the engine in an hour is $60 \times 33{,}000$ foot-pounds, the efficiency, which includes the boiler, is therefore about 14 per cent. In this case the determination of the efficiency requires that the energy given off in foot-pounds on the engine shaft must be converted into equivalent thermal energy, so that it may be compared with the energy in heat-units caused by the combustion of the coal supplied to the boiler, or the heat energy in the coal might be expressed as mechanical energy, giving the same result. The efficiency of the steam plant is also made up of the combined efficiencies of the boiler, pipe line, and engine, and, as engineers well know, it is frequently convenient to refer to the efficiency of the

steam engine alone by an arbitrary standard of the number of pounds of water per hour in the form of steam at a certain pressure that are necessary to give a horse-power hour in mechanical energy on the shaft. Such computations are simpler for water power, for the efficiency of a water wheel is directly found, and does not depend upon what is generally an assumption in the case of the steam or internal combustion engine—the calorific value of the fuel. The weight of the water passing through a wheel, and the height through which it has fallen being known, the power supplied is directly ascertained. As the efficiency of water wheels and wind engines will be dealt with elsewhere we may conclude by the statement that of all prime movers the water turbine is the most efficient machine that we possess, as it will convert fully 80 per cent. of the energy supplied to it into useful work, the internal combustion engine comes next with a performance of about 40 per cent., and lastly the steam engine and wind engine which rarely exceed 20 per cent., and are usually very much less. If electric motors were included in the category of prime movers, they would take first place, as the efficiency of large machines is well over 90 per cent., but as they cannot be described as prime movers, they cannot properly be classed in this connection. Nearly all the inventions and improvements made since prime movers were first introduced have been directed towards an improvement in efficiency, and with conspicuous success in the case of hydraulic machinery, though not less so in the mechanism of the steam engine, but the great heat losses in the boiler will, as far as it is now possible to foresee, always keep the efficiency of the steam plant low. Notwithstanding the

use of steam turbines fitted with condensers, together with the various adjuncts to the steam plant, such as economisers, feed water heaters, and superheaters, the efficiency of the whole transformation process from heat to mechanical energy by this means is very low.

CHAPTER II.

WATER POWER AND METHODS OF MEASURING.

The Physical Properties of Water.

For a practical understanding of the subject of water power as known and applied by the engineer in his work, it is not necessary to step beyond the elementary principles of hydraulics and hydrodynamics. The rough-and-ready rules which are applied to problems met with in practice, many of which are to be found in handbooks, rest upon a few general principles in hydraulics, easily understood and easily applied. Notwithstanding the great fascination which the broader subject of hydrodynamics has for the student, as expounded by Professor Lamb[1] and others, the engineer who is called upon to apply principles rather than follow lines of absorbing investigation must perforce be content at leaving off where the real student of hydrodynamics begins. It is up to this point that the reader is taken in the following pages, and the principles contained in them are those which the hydraulic engineer is called upon to apply in his work.

It is natural that the subject of hydraulics should open with a few facts concerning the physical properties of water. Let us first consider the density or weight of the

[1] Treatise on the Mathematical Theory of the Motion of Fluids, Professor Lamb.

fluid. Unlike most other substances which vary in density in one direction with the temperature, water attains a maximum density at 4° C., above or below which temperature the density diminishes, but to such a small extent as to be negligible to the engineer, but important to the physicist who is carrying on refined experiments. The following table shows this, in which the relative volumes at different temperatures are given[1]:—

t °C.	Volume (Rossetti).	Wt. of 1 c.c., in grammes.
0	1·000129	0·999884
4	1·000000	1·000013
100	1·04312	0·95866

The unit weight of water will be taken to be 62·5 lbs. per cubic foot. Although the weight of a cubic foot of distilled water at the ordinary temperature is 62·4 lbs. nearly, impurities both dissolved and suspended somewhat increase the specific gravity, so that 62·5 lbs. may be used in what follows as the weight in British units, while in the metric units the cubic centimetre of pure water at 4° is, by definition, the gramme. The simplicity of the metric system for hydraulic calculations is especially apparent, inasmuch as

[1] The behaviour of water under enormous pressure such as that developed in the explosion chambers of modern artillery has formed the subject of recent investigation at the hands of a skilled engineer. It is asserted that under pressures of about 20 tons per square inch the fluid can be reduced in volume by several per cent. This does not do violence to the common and accepted hypothesis that water is an incompressible fluid, for under the ordinary pressures at the surface of the earth, and those with which the engineer is ordinarily familiar, it may be regarded as such. The sea-water upon the bed of the Caribbean Sea is exposed to a pressure of about 5 tons per square inch.

the kilogramme of water and litre represent the same quantity of water. The only logical measure in the British system is the gallon, which weighs 10 lbs., but the universality of this is unfortunately destroyed by the fact that the United States gallon weighs 8·34 lbs., and therefore there is a likelihood of confusion. For this reason the gallon will be omitted, and where British measures, in contra-distinction to metric, are employed, the cubic foot shall be the chosen unit of volume for water, for it is generally used by hydraulic engineers as being more convenient than the gallon for calculations.

The Power in Falling Water.

It has already been pointed out how the horse-power is defined, and that it represents 550 ft. lbs. (or the equivalent in metric units of 76·08 kgms.) exerted during one second or, what is more correct, energy expended at this rate, for it is of course unnecessary that the expenditure of energy shall be continued for a second; all that is necessary to define the power is that the *rate* is a certain value. The value of the horse-power in metric using countries is 75 kgms. per second, which is slightly less than the English and American unit. The French force de cheval and the German Pferdekraft have this value. When using the metric system it will be convenient to refer to this unit, as the difference is less than 2 per cent. between it and the horse-power of 550 ft. lbs. per second. If, therefore, 75 litres of water fall through a distance of one metre every second, the fall is capable of yielding up continuously one horse-power, and if it were possible to convert all of this to our use by machinery, we would be reclaiming all

that is possible from such conditions of flow and height of fall. To express the power of a fall generally, where γ is the weight of water falling per second, and h the height through which it falls—

$$\text{H.-P.} = \frac{\gamma \times h}{550} \text{ or } \frac{\gamma \times h}{75}$$

according as γ and h are expressed in British or metric units. For example, a stream, as found by measurement, discharges 118 cu. ft. of water per minute, which falls through a distance of 9·25 ft., the horse power is therefore—

$$\text{H.-P.} = \frac{118 \times 62{\cdot}5 \times 9{\cdot}25}{33{,}000} = 2{\cdot}1.$$

If, again, the discharge necessary to develop a given horse-power falling through a known height be required, it may be found as in the following case. Sixty horse-power is required from a fall of 9·37 metres, the discharge (d) per second in litres is therefore—

$$d = \frac{60 \times 75}{9{\cdot}37} = 480 \text{ litres} = 0{\cdot}480 \text{ cu. metres.}$$

As the efficiency of the water turbine may be taken as 75 per cent., for rough calculations the following simple rule for obtaining the net horse-power of a fall in metric units is convenient—

$$\text{H.-P.} = \frac{\gamma \times h}{75} \times 0{\cdot}75 = \frac{\gamma \times h}{100}.$$

Thus, by multiplying the litres per second by the height of the fall in metres and dividing the quotient by 100, the actual horse-power which is developed at a turbine shaft may be obtained. This rule is, of course, only

approximate, involving as it does the assumption of an efficiency of 75 per cent. for the wheel. In the above case—

$$\text{H.-P.} = \frac{480 \times 9{\cdot}37}{100} = 45.$$

From this it is evident that fifteen horse-power is absorbed by the friction of the water passing through the wheel and connections, and other losses.

The following are convenient to remember in this connection :—

(1) One horse power is developed by 8·8 cu. ft. per second falling 1 foot.
(2) ,, ,, ,, 75 litres per second falling 1 metre.
(3) ,, ,, ,, 1 cu. ft. per second falling 8·8 feet.
(4) ,, ,, ,, 1 litre per second falling 75 metres.

A continuous stream of water is essential if the power be required for driving water wheels, and the measurements which the engineer is called upon to make to ascertain the extent of the power at his disposal, involve the measurement of the flow, and the vertical height through which the water passes, or the estimated difference which will exist between the level of the water in the head and tail races when the works are completed.

Velocity of Falling Water.

It would seem almost unnecessary to call to the mind of the reader the relationship between the velocity of a falling body and the height through which it has fallen, but through this relationship the hydraulic engineer is enabled to reduce his calculations to simple formulæ, and to speak of "head" of water, as a mechanical engineer, specifying the qualities of a boiler, would speak of the "pressure" it was designed to carry. If, then, a body be allowed to fall

freely through a height h, it will acquire a velocity v, which bears the following relationship to h :—

$$v = \sqrt{2\,g\,h} \text{ or } h = \frac{v^2}{2\,g}.$$

The symbol g stands for the acceleration due to gravity, the value of which may be taken as 981 cms. or 32·18 ft., per sec. per sec., v being correspondingly expressed in centimetres per second or feet per second. For instance, the theoretical velocity of water falling through a distance equal to the height of Niagara (156 ft.) would be—

$$v = \sqrt{2 \times 32{\cdot}18 \times 156} = 100 \text{ ft. per sec. approx.} = 68 \text{ miles per hour.}$$

The curve (Fig. 1) is plotted with heights of fall in feet as abscissæ, and velocities as ordinates, the latter being calculated by the foregoing formula. For example, a fall of 25 ft. corresponds to a velocity of about 40 ft. per second. The straight line shows the relation between height of fall and horse-power for one cubic foot of water per second. For example, one cubic foot per second falling through 70 feet is seen to give about 8 horse-power.

No account is taken of air resistance in this formula, or of the loss in velocity which inevitably occurs when water is constrained to move in pipes or closed channels. These losses in practice are considerable, and will be treated of later, but at present we are only concerned with the conditions prevailing where nothing exists to disturb the action of gravity, not even air resistance. A stream of water is rapidly broken up in falling through the air, and in the case of some high falls there is nothing but spray and moss-

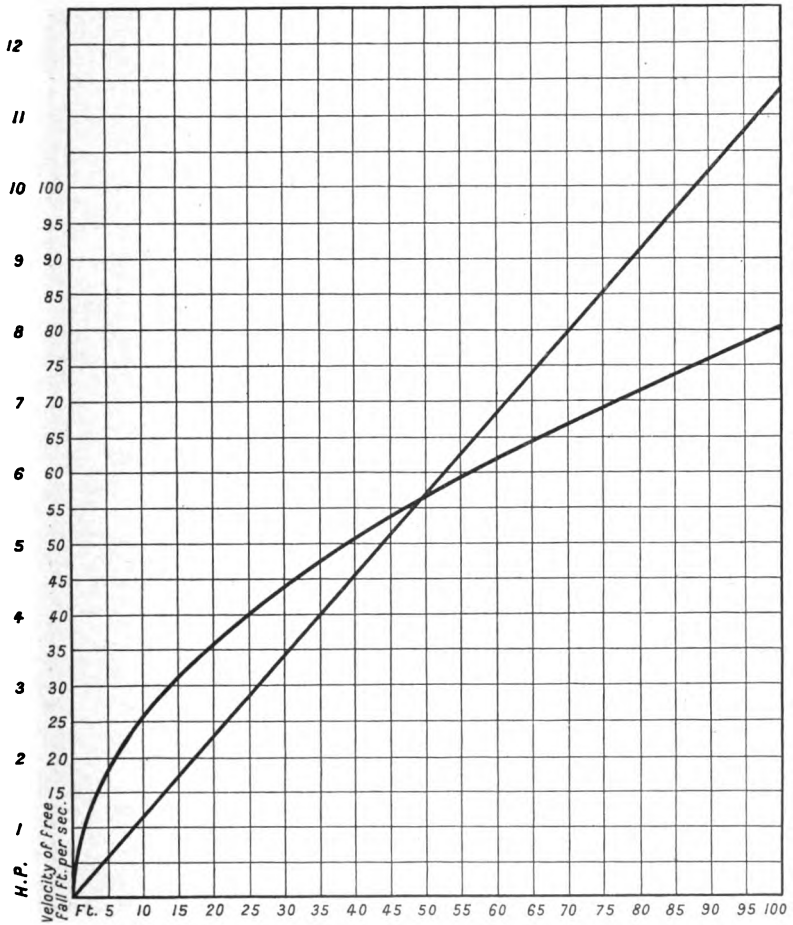


FIG. 1.—Relation between height of fall and velocity of water.

covered rocks to testify to the existence of a stream tumbling over a ledge some hundreds of feet above, to which phenomenon the tourist from the Grand Canyon of the

Colorado can bear witness. The velocity of the falling water, dispersed into small globules or spray, may thus be reduced indefinitely by the action of the air or arrested altogether, and the speed of falling raindrops has formed the subject of mathematical investigation of a very recondite order. Inasmuch as the engineer who is contemplating the utilisation of the waterfall, lays his plans so as to confine the water for his purpose, the formula as given above may be applied to estimate the potentiality of the stream, or as a basis upon which to build his hopes of a financial return, coupled with the results of his measurements upon the volume of water which the stream discharges.

The relation between head and velocity being expressed for a free fall, the one as a function of the other, it is clear that when one is known the other is easily ascertainable. Engineers are accustomed to allude to "velocity head" when, by examination, they have discovered the velocity corresponding to a certain height of free fall. The mean velocity of the water across a section of a pipe is, let us say, 17·3 metres per second, consequently the velocity head is—

$$h = \frac{(17{\cdot}3)^2}{2 \times 9{\cdot}81} = 15{\cdot}25 \text{ metres.}$$

This does not necessarily mean, and in practice it never does mean, that the open water level is 15·25 metres above that section of the pipe where the velocity is measured. The actual head is, in fact, greater than the velocity head, for reasons which will be made clear by a full examination of the theorem of Bernouilli.

The Theorem of Bernouilli.

This theorem, which was first demonstrated by the Swiss mathematician and physicist, Daniel Bernouilli, in 1738, covers all the problems which arise in the investigation of the flow of water in pipes, and very important principles and fundamental truths are set forth in the few symbols into which the comprehensive principles are compressed.

The theorem may be stated in the following concise terms. If p_a and v_a be respectively the pressure and velocity of the water in a closed pipe at a section A, and h_a the height of the section above a given horizontal line, and p_b, v_b, and h_b be the corresponding quantities at a section B, then

$$(1) \qquad \frac{p_a}{\gamma} + \frac{v_a^2}{2g} + h_a = \frac{p_b}{\gamma} + \frac{v_b^2}{2g} + h_b,$$

assuming that water is a perfect liquid, and that it is flowing through a frictionless pipe. But $\frac{p}{\gamma}$ is the head due to the pressure at the point under observation, *i.e.*, it is the height of a column of the liquid that could be supported at the point by the pressure there, also $\frac{v^2}{2g}$ is the further height to which the liquid would rise in virtue of its velocity, if suitably guided upwards, while h is the elevation which it already possesses, so that the sum of these three quantities above the datum chosen represents the height which it is capable of reaching by reason of all three, and the theorem expresses the fact that this is the same at all points in the pipe running full of water.

Assume that there is a pipe (Fig. 2) leading out from an

open reservoir filled with water, and that by some arrangement for admitting water, the level is kept constant when water is drawn off at the bottom by the opening of the valve B. If h_1 is the height of the water level above a datum chosen for convenience below the valve, then the

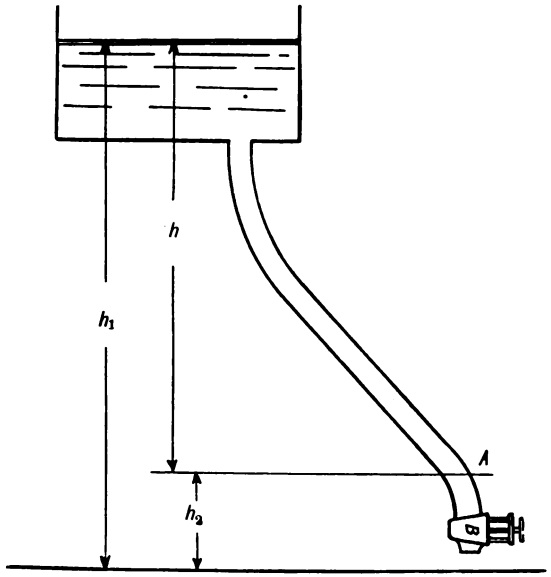


FIG. 2.—Reservoir and pipe to illustrate the theorem of Bernouilli.

head of water above a section A is $h = h_1 - h_2$, and there will be a static pressure in the pipe at the section due to the weight of water above it, when the valve B is closed and the water is still. The head h producing this pressure is conveniently referred to as the pressure head, and the pressure per square unit upon the walls of the pipe at A is $p = \gamma h$, where γ is the weight of a cubic unit of water, and h the head in corresponding units.

If in equation (1), the suffix a refers to the point A on the pipe, and b to the surface of the water in the reservoir, then, since v_a and v_b, are each equal to zero, and p_z is the atmospheric pressure, we have

$$\frac{p_b}{\gamma} + h_b = \frac{p_a}{\gamma} + h_a, \text{ or } h_b - h_a = \frac{p_a - p_b}{\gamma}$$

but $h_b - h_a$ is the static head of water on the section A, and $p_a - p_b$ is the pressure at that point above the atmosphere, so that the theorem shows the truth of the simple law giving the relationship between static head and pressure of water.

Now suppose that the pipe is open to the atmosphere at A for the water to discharge, and that the water level in the reservoir is kept constant by a supply equal to the discharge, then since $v_b = o$, and $p_a = p_b$ (since at the surface and at B the pressure is atmospheric) we have

$$\frac{v_a^2}{2g} + h_a = h_b \text{ or } h_b - h_a = \frac{v_a^2}{2g}$$

but $h_b - h_a$ is, as before, the head above the point A, and this head is now utilised in imparting kinetic energy to the water and the expression $v_a^2/2\ g$ may be referred to as velocity head. The discharge of the pipe into the atmosphere at A would be the velocity multiplied by the area of the pipe at that point.

As an example, supposing $h_b - h_a$ be 26·2 feet and the diameter of the pipe be uniformly 6 inches. The discharge, neglecting friction, would be

$$Q = A \times \sqrt{2g(h_b - h_a)} = \frac{\pi}{4} \times \left(\frac{1}{2}\right)^2 \times \sqrt{2 \times 32{\cdot}2 \times 26{\cdot}2}$$

= 8·05 cu. ft. per sec.

The foregoing are two extreme conditions employed for the purpose of illustration, but in practical problems the head is always divided between velocity and pressure head along a line of pipe, and the relation between the two is constantly varying as different levels are reached and different sizes of pipes are encountered in a system.

Thus when a pipe, running full of water, is decreased in diameter at a point, the velocity is increased and the pressure declines. This principle is made use of in the Venturi meter which is an instrument for measuring the quantity of water flowing along a pipe, and which is extensively used in water supply systems, but is not employed in connection with water power plants. The pressure in a pipe may be less than the atmospheric as the above equation shows, but in service pipes this is not often the case.

The effective head, which is constantly referred to by hydraulic engineers, is the head corresponding to the total energy that may be obtained at the turbine or other apparatus which is to be driven by the water. In other words, the effective head is the actual head, as determined above, less the losses which occur owing to the flow of the water, and which must therefore be deducted, as they reduce the total energy available at the place where it is required. Though this is the exact definition of effective head, the term is often incorrectly applied, as for instance to the actual difference in elevation between the head and tail water of a waterfall, which cannot all be utilised.

But so far the problem has been simplified by the omission of the losses which are always present to the engineer, and which considerably modify the results. These losses of energy in flowing water are caused by

the friction in the pipes, eddies at bends and round obstacles such as rivet heads or badly fitted spigots, also losses due to the friction of the particles of water between themselves owing to the fact that all the particles do not move with the same velocity. Again, there is considerable loss due to sudden reductions in the velocity, such as occur when a pipe is suddenly enlarged. The reduction of these losses to a minimum, and an accurate means of estimating their magnitude when laying out new work form an important part of the subject of hydraulic engineering.

By an examination of the foregoing equation it is seen that the pressure to which the pipe is subjected at a section cannot, under conditions of uniform flow, be greater than that due to the head of water above the section in question, and that it is greatest when there is no flow in the pipe, *i.e.*, a static head. There is, however, a cause hereafter to be considered which may give rise to great momentary increase in the pressure, but at present the acceleration or retardation of the water in the pipe are left unconsidered, and we are dealing with uniform velocity throughout the pipe. Generally the diameter of pipes vary throughout a system, but where all the pipes are running full $a_1 v_1 = a_2 v_2 = \text{const.}$, where a_1 and a_2 are the areas of the pipe at two places and v_1 and v_2 the velocities across the sections at these places. This relation is referred to by some writers as the equation of continuity and is self-evident, and as the areas are proportional to the squares of the diameters, the relation may be written $d_1^2 v_1 = d_2^2 v_2$.

An example to illustrate the preceding facts may be drawn from practice. A pipe carrying water down a mountain

side to a turbine installation is 2 ft. internal diameter, and the water level in the reservoir is 420 ft. above the valve at the bottom. Again, assuming the losses to be small even when the turbine is working at full gate with a velocity of 7 ft. per second through the pipe, we will proceed to find the value of the velocity head for various velocities up to 7 ft. per second. This is shown by the following table, where the total head of water is variously divided between pressure head and velocity head.

Velocity in pipe ft. per sec.	Velocity head in feet. $\frac{v^2}{2g}$	Pressure head in feet. $\frac{p}{\gamma}$	Pressure lbs. per sq. in.
0	0	420	182
1	0·015	—	—
2	0·062	—	—
3	0·14	—	—
4	0·25	—	—
5	0·39	—	—
6	0·58	—	—
7	0·76	419·24	181·67

The velocity of the water through the pipe when at the maximum means a slight reduction in the pressure of about 0·33 lbs. per square inch, which is only about 0·17 per cent. of the pressure due to the static head, and except for illustrating the principle, is of little value practically, as the pipe has to be designed for strength to withstand higher pressures than the static head.

Weir Measurements.

The simplest method of obtaining an accurate estimate of the quantity of water passing down a small stream is by means of a weir over which the water is allowed to fall (Fig. 3).

The weir as usually used by hydraulic engineers consists of a flat panel set vertically across the stream out of which a rectangular notch is cut through which the water flows. The height of the water over the weir to be used for calculating the flow is measured from the sill of the weir to the surface of the stream some feet above the weir, as, owing to the increased velocity of the water at the edge of the weir the surface at that point is depressed. Two precautions are necessary in making such a measurement: (1) To be certain that the sill or edge of the weir is horizontal. (2) To measure the head as aforesaid by means of a suitable gauge.

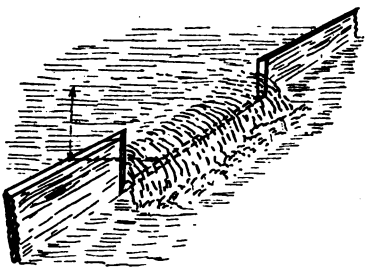


FIG. 3.—Weir for measuring the flow of a stream.

If the stream is too wide to permit the use of a weir, or the banks render access for such a purpose impossible, the next best way of obtaining the flow is by taking soundings across the section of the stream, and obtaining the superficial area of the cross section, then by measuring the velocity by floats, the amount of water passing is obtained by taking the product of the cross section and the velocity expressed in the same units. The velocity of the water at all points across the section is not the same. The friction against the sides and bottom reduce the velocity somewhat, so that a float, carried down on the surface, would indicate a higher velocity than the true average for the whole cross section. The average velocity may be more closely attained by means of a loaded stick floating

vertically, or a bottle which projects downward into the current beneath the surface.

In all important hydrometric investigations velocity meters are used which may be lowered from a boat to any point, and thus from a series of such measures the mean velocity and flow can be calculated. It would take us too far to describe the various kinds of velocity meters and other appliances used for these measurements in large rivers, and to follow the reduction of the observations to the desired result. These current meters, which consist essentially of a small screw propeller with means for measuring the revolutions when it is immersed in a current of water, have to be carefully calibrated experimentally in order to determine the relation between the revolutions of the screw and the speed of the current. Some experiments were made in a tank at Haslar by Mr. R. Gordon, the results of which [1] show that the speed of the meter and that of the current are not directly proportional. Thus, for a speed of 1 ft. per second a certain meter makes 0·82 revolutions, while in a current of 4 ft. per second it makes 3·4 revolutions in the same time. In general it is expedient to obtain the constants and errors of current meters over a wide range of values, and to plot a curve from which the speed of the current may be directly read off when the revolutions of the meter are given. These are usually to be obtained from the manufacturers who determine them by dragging the meter through water at different velocities and noting the revolutions of the counter. Weir measurements constitute the most reliable guide to the flow in small rivers and streams, and among the

[1] "Proceedings Inst. M. E.," May, 1884.

investigations that have been carried out, those of Mr. J. B. Francis are most noteworthy. He has left us most complete records of experiments which were undertaken upon the flow of water over weirs, and they have so far not been surpassed in accuracy. While there is a variety of choice of weir formulæ open to the engineer, the Francis formulæ give results that cannot be gainsaid, and the weir tables and curves (Fig. 4) are based upon these formulæ. The reason for the differences that exist between the results of different experiments is that the coefficient of flow has not a fixed value, and various assumptions are made for it.

If h be the height of the surface of the water above the sill of a weir, and b the breadth of the weir in feet, the theoretical flow (Q) in cubic feet per second is

$$Q = \frac{2}{3} b \sqrt{2 g h} \times h = 5{\cdot}35\, b \sqrt{h^3}.$$

This would be the flow if water was a perfect fluid, but actually it is less than that given by this relation, by a certain amount depending upon the coefficient of flow. This coefficient is variously estimated and this gives rise to numerous weir formulæ according to the value assigned to it. Moreover, it is not a constant for the same weir, but varies with the height of water above the sill. This variation is noticed in the formulæ used by some engineers, but for small weirs with heights up to 2 ft. a sufficient degree of accuracy is attained by assuming a mean value for the coefficient. The formula of Braschmann, which is much used in Switzerland, takes account of the variation in the coefficient, but it is generally the case that in the ordinary course of the measurement of flow of small rivers and streams the refinement of a varying

coefficient is unnecessary, as it represents a questionable degree of accuracy. The writer has used Francis' formula as being the simplest and most convenient for small weirs, and accurate for all conditions that arise in practice.

The coefficient, or ratio of the actual flow to the theoretical, adopted by Francis, is 0·62, so that the formula, expressed in English units, is

$$Q = 0{\cdot}62 \times 5{\cdot}35\ b\ \sqrt{h^3} = 3{\cdot}33\ b\ h^{\frac{3}{2}}.$$

To put it into a more convenient form, let d be the width of the weir in inches, h, as before, the height of the water over the sill in feet, and Q the flow in cubic feet *per minute*, then we have

$$Q = 16{\cdot}65\ d\ h^{\frac{3}{2}}.$$

The curve (Fig. 4) is plotted from this formula.

If d and h be expressed in centimetres and Q in *litres per minute*, the equation becomes[1]

$$Q = 1{\cdot}102\ d\ h^{\frac{3}{2}}.$$

The discharge shown on the curve is plotted from this formula.

The vertical ends of a weir deflect the passing water so that the width of the stream immediately below the weir is not equal to the weir width. This deflection is caused by the angularity in the approach of the water when the weir

[1] The transformation from English to metric units is effected by substituting the values for cubic feet, inches, and feet, in litres and centimetres—

$$\frac{Q}{28{\cdot}3} = 16{\cdot}65 \times \frac{b}{2{\cdot}54} \times \frac{h^{\frac{3}{2}}}{30{\cdot}48 \times \sqrt{30{\cdot}48}}$$

or

$$Q = 1{\cdot}102\ b\ h^{\frac{3}{2}}.$$

does not extend across the full width of the stream. Francis allows for these end contractions in a modification of his formula, but unless the weir is less than 3 ft. wide the correction for the end contractions may justifiably be left out, and even for a less width the error by leaving it out is small.

WEIR TABLE.

Depth on Weir (h) in inches.	Cubic ft. per min. per inch length.	Depth on Weir (h) in inches.	Cubic ft. per min. per inch length.
1	0·4	10	12·7
1·5	0·7	10·5	13·7
2	1·1	11	14·6
2·5	1·6	11·5	15·6
3	2·1	12	16·7
3·5	2·6	12·5	17·7
4	3·2	13	18·8
4·5	3·8	13·5	19·9
5	4·5	14	21·1
5·5	5·2	14·5	22·1
6	5·9	15	23·3
6·5	6·6	15·5	24·5
7	7·4	16	25·7
7·5	8·2	16·5	26·9
8	9·1	17	28·1
8·5	10·0	17·5	29·4
9	10·8	18	30·6
9·5	11·7	18·5	31·9

The accompanying tables and curves give the flow over a weir for a unit width. It should be noted that the quantity passing a weir of only 1 in. or 1 cm. in width would be somewhat less than that given by the table or curve, owing to the end contractions in such a case having a relatively great influence, but as it is seldom that a weir would be anything like as narrow, the flow *per inch width* or *per centimetre width* for such a weir is correct.

Example: A weir is 4 ft. 8 ins. wide and the height over the sill is 10 ins. The flow in cubic feet per minute is therefore $12{\cdot}7 \times 56 = 711{\cdot}2$ cu. ft.; or again, a weir 1·46 metres wide with a depth of water of 36 cms.

Depth on Weir (h) in centimetres.	Discharge in litres per min. per centimetre length.	Depth on Weir (h) in centimetres.	Discharge in litres per min. per centimetre length.
1	1·10	26	146·5
2	3·12	27	155
3	5·74	28	164
4	8·84	29	173
5	12·4	30	182
6	16·2	31	191
7	20·5	32	200
8	25·0	33	209
9	29·8	34	219
10	34·9	35	229
11	40·3	36	239
12	45·9	37	249
13	51·8	38	259
14	57·9	39	269
15	64·2	40	280
16	70·7	41	290
17	77·4	42	301
18	84·4	43	312
19	91·5	44	322
20	98·8	45	333
21	106·2	46	345
22	114·0	47	356
23	121·8	48	368
24	129·8	49	379
25	138·0	50	390

gives a flow in litres per minute of $239 \times 146 = 34{,}894$ litres $= 34{\cdot}894$ cubic metres per minute.

It is important to obtain the height of the water over the sill some few feet up-stream to avoid the declivity of the surface owing to the velocity of approach. The hook gauge, as used in laboratories, is a very delicate means of

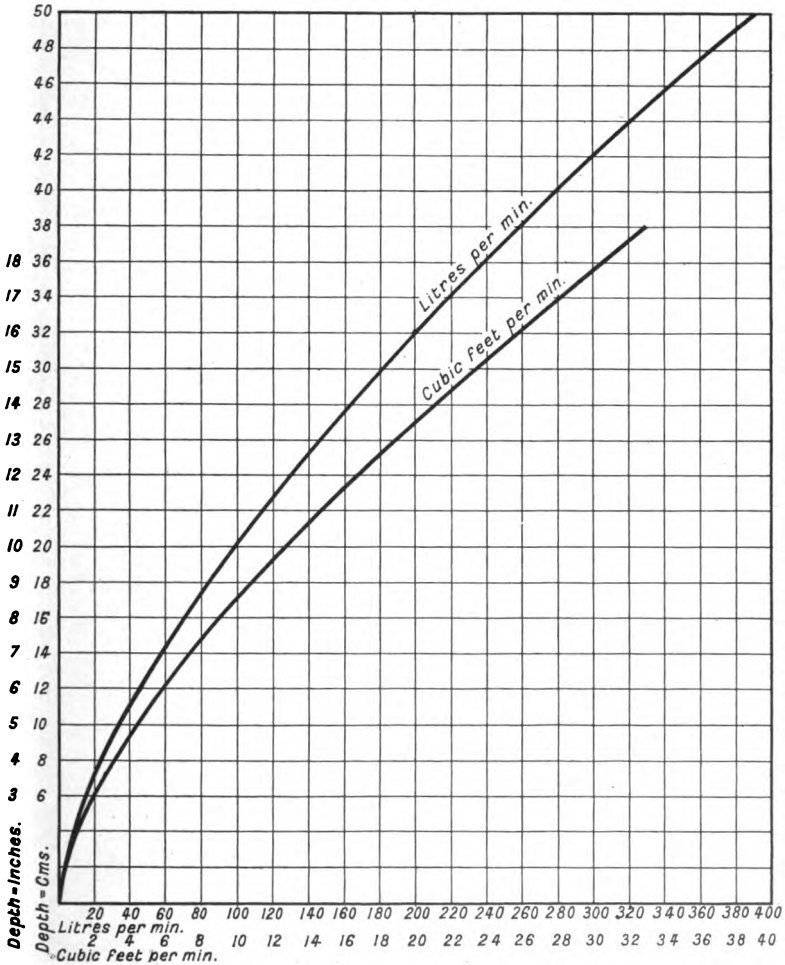


FIG. 4.—Discharge over weirs.

measuring the height. The gauge consists of a wire turned up at the end in the form of a hook and pointed.

The hook is immersed and is adjustable by means of a screw and nut. The screw is turned until the point of the hook reaches the surface and the reading is then taken which is referred to the sill of the weir. A two-foot rule used a few feet up-stream is generally as accurate as occasion warrants. If the surface of the water be broken the hook gauge is troublesome. If, however, it is used in such cases it can be protected from disturbance by sinking a pipe inside which the gauge may be placed.

Small streams with variable flow should be measured during the driest weather when the water passing may be taken to be the least throughout the year, and the plant may then be designed with a view to utilising this minimum flow in one wheel, so that it will be working at as near full load as possible, and therefore at the highest efficiency. Violent changes and fluctuations in the flow from hour to hour, which occur on small watercourses in rainy districts, are the most difficult to deal with, unless there is sufficient storage capacity above the fall to allow a steady discharge to be maintained. The effect of a heavy flood on such streams is often to diminish the effective head on the wheels by raising the tail water, and this may have the effect of shutting down the plant altogether, which may be serious if there be no substitute for furnishing the power or light.

Levels and Levelling.

A preliminary survey of a site for a proposed water-power plant should begin with a measurement of the height of fall available, allowing for the possible reductions at all stages of the river. If there be a natural waterfall

the height may readily be measured from surface to surface by means of a plumb line, but if the development of the water power necessitates the construction of a dam or canal by which the fall is obtained, it is necessary to "run a line of levels" to ascertain the fall in the river from point to point, and by this means determine the height which the dam should be, or the length of canal

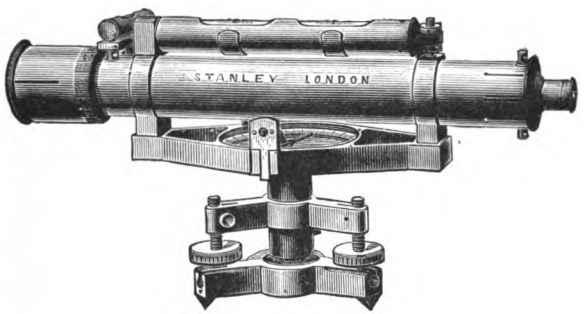


FIG. 5.—The engineers' level.

necessary to obtain the desired difference of elevation at the site of the proposed water wheel installation. The levelling is usually carried out by an engineers' level, an instrument which, in the hands of a skilled craftsman, will yield very accurate work. An ordinary level, such as that shown in Fig. 5, which is employed for the purpose of railway levelling, will, if in proper adjustment, yield a very accurate result in skilled hands. As an actual example, a line of levels was run for three miles which showed a difference of elevation between the two ends of the three-mile stretch of about 120 ft. The levelling was then repeated backwards with the result that there was only a difference of 0·75 ins. between the two results, or about 0·05 per cent.

difference. Such a result is only possible by great care on the part of the operator, and would not be attained without considerable practice. For hydraulic measurement a much less accurate result would be satisfactory if the object was merely to ascertain the height of fall, which is subject to such continual variation.

The engineers' level consists of a telescope provided with

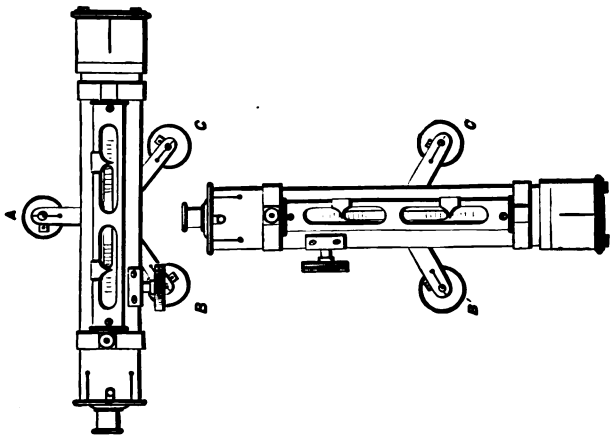


FIG. 6.—Adjusting screws for levelling.

a focusing arrangement and cross-hairs, the intersection of which mark a point in the field of view. Attached to the telescope and parallel with it is a spirit-level, so that when the bubble is in the centre of the tube, *i.e.*, at the highest point, the telescope is horizontal. Levelling screws are provided for altering the inclination to the horizontal by which the telescope may be levelled. The whole arrangement is mounted upon a vertical axis about which it turns, and the instrument is supported by a tripod. It is

essential that the level be properly adjusted before being put into use. The principal adjustments are (1) The parallelism between the optical axis of the telescope and the level. (2) The perpendicularity between the telescope and the vertical axis. This latter relation cannot usually be altered by the user, as the instruments leave the factory with the relation fixed, but the level is provided usually with lock nuts by which adjustment (1) may be easily effected.

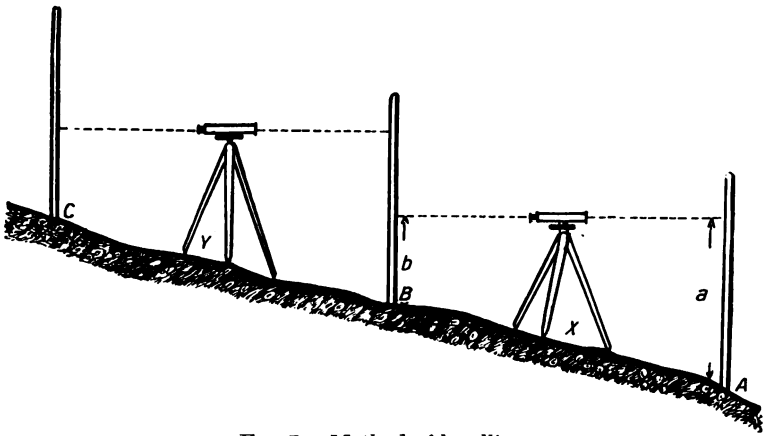


FIG. 7.—Method of levelling.

The level proper is a glass tube, the upper surface of which is curved to a circle of several feet radius. It is filled with pure alcohol or a mixture of alcohol and sulphuric ether, and the bubble at the top spreads out over some divisions scratched on the glass, by which the position of the bubble relative to the centre line may be seen. The instrument is brought to a level when the tripod is planted, by adjusting three thumb-screws (Fig. 6). As a plane surface will always be level when two lines at right

angles upon it are level, the most expeditious way of levelling is to level one line parallel to the line joining two of the screws and another line at right angles to this. When this is done, if the level be in proper adjustment, any position of the telescope when rotated about the vertical axis will be level—as shown by the unchanged position of the bubble. The actual process of levelling is illustrated in Fig. 7. It is desired to obtain the difference in height between two points A and B. The level is set up at any intermediate point, not necessarily between or in line with A and B. After the instrument is levelled, the observer reads the height a upon the graduated rod which is held by his assistant vertically upon the point A. This reading having been recorded, the assistant transfers the rod to the point B, where the reading (which is usually taken to one-hundredths of a foot) is also recorded. If the first reading a be 8·27 ft. and b be 4·87, it is clear that the difference in height between A and B is $8{\cdot}27 - 4{\cdot}87 = 3{\cdot}40$ ft. The height of the instrument above the ground is not a factor in this case. Any convenient height will do, since it is the difference of the readings that denotes the difference in elevation of the two stations. If, however, the difference in elevation is required between points which are too distant from each other to be sighted from one setting of the instrument, it is necessary to accomplish the levelling by a series of steps which are known as a line of levels.

The telescope is of such power in the ordinary engineers' level that the small divisions on the rod may be discerned up to 250 or 300 ft., beyond which it is not possible to ensure accuracy, and therefore distances of more

than 600 ft. must be covered by more than a single setting of the instrument. The method of taking a series of levels and recording the results is the same as for a single pair of stations, but some recognised plan for keeping the notes is necessary to avoid confusion. The diagram shows two stations A and B with the instrument levelled, from which readings *a* and *b* are obtained. These two readings are set down in the level-book in columns 2 and 3. They are known as backsights and foresights, as, in the first reading the level is turned back upon a station while in the second it is turned forward to read the rod when placed upon an advanced station. It is also evident from the diagram that *a*, the backsight reading, is the height of the instrument above A, and this is frequently set down in level books as H. I., being the height of the instrument above the station to which a backsight is directed when the instrument is set up. A page of a level-book would appear thus :—

1. Station.	2. H. I.	3. Foresight.	4. Elevation.
A	8·27	—	0
B	7·25	4·87	3·40
C	—	3·49	7·16

The first column gives the name of the station, and H. I., the backsight reading on the first station is entered as 8·72 in the second column. The elevation of this first station may be any arbitrary figure. Sometimes it is referred to sea level when its elevation is known, and is expressed in feet. If the elevation is not known it may be entered as 0.

The foresight reading on B, which is 4·87, is set down in column 3 in the line opposite B, and the elevation of this station, being 8·27 — 4·87 = 3·40, is placed in column 4. The instrument being removed from the first position at X to a new position Y, a backsight is taken upon B which is the new height of the telescope above B. This reading is found to be 7·25. The rod is now placed at C and the foresight is 3·49. C is therefore 7·25 — 3·49 = 3·76 ft. above B and 3·40 + 3·76 ft. above the datum; therefore its elevation is 7·16. It will be seen that by adding up all the figures in column 2 and subtracting the sum of those in column 3, the elevation of the last station is directly obtained. In the example cited the last station is higher than the first and the ground rises steadily. If however it is surmised that the line of levels will be taken downhill, as when they are run in a down-stream direction on a river bank it is convenient to assume an elevation to start with so that negative signs may be avoided.

It is immaterial whether the backsights and foresights be equal in length or different unless the operator has reason to believe that the instrument is out of adjustment by not having the optical axis exactly perpendicular to the vertical pin upon which the level swivels. If there is an error in this, the backsights and foresights ought to be as nearly as possible of equal length, for by so doing this error is diminished, and if exact equality could be attained it would be entirely eliminated. As this source of error in a level is not easily put right in the instrument—and levels subject to rough usage are liable to become deranged—it is safest for accurate work to take both sights as nearly as possible of equal length, and this is generally done on long lines of levels

where a cumulative error might make a great difference from end to end of the line if, as is often the case, the line be some miles in length. Accuracy in the use of a level depends mainly upon the skill of the craftsman, who must know how to make the various adjustments, and the errors or bias of his instrument. The three legs, which are shod with metal, should be firmly planted in the ground and then, by means of the screws, the telescope brought to an exact level. The manipulation of the screws on a cold day is a trying experience, particularly when they are stiff, as they ought to be. By repeatedly turning the telescope through angles of 90° and correcting by the screws until no change of position in the bubble is observed for any position of the telescope, the instrument is prepared for the first reading.

The rod in the hands of the rod-man should not be held rigid, but should be balanced upon the selected point on the ground, and sustained by the tips of the fingers on each side. By such means a vertical position of the rod is assured. Some practice on the part of the rod-man is necessary before he can balance the rod, especially in a high wind. It is usual to mark the stations upon a short survey by wooden pegs driven into the ground, upon the tops of which the rod rests when taking a reading. They also serve the useful purpose of preserving a record of the stations so that check levels may afterwards be made should there be any doubt about the accuracy of the first, or a desire to confirm the previous figures. As some stations may be placed at conspicuous or definite points of which the elevation is required, some means of permanently marking them is essential. Most of the stations or turning

points, however, are merely links in the chain of levels, and their actual elevation is not a matter of note. If, however, the elevation of intermediate points be required, a stone or other solid rest for the rod may be used—anything, in fact, that will afford a rigid base during the time the man at the instrument can take a fore sight, move the level to a new position, and take a back sight, after which the rod-man goes forward to give the leveller a fore sight upon the next station.

It is customary in taking levels along a river to select convenient points or bench marks of a permanent kind so that the elevation of the surface of the water may always be found at all stages of the river by reference to them. A notch at the base of an extended root of a tree, or a marked position upon a rock or firmly embedded stone, may be conveniently used as a bench mark from which the level at all stages of the river may be obtained, and if there are several marks of known elevation along the bank, the variation in the fall of the river at different stages of flow may be readily observed. This is especially important owing to the effect which heavy floods have on small rivers of diminishing the head, an example of which, at the site of a dam on a small river, is referred to elsewhere. With a heavy flow the tail water in this case was raised so as to seriously diminish the available head which the dam was designed to afford (see page 136). In this case bench marks of known elevation were fixed above and below the dam in such positions that the distance down to the surface of the water from each could be directly obtained by readings on a rod, and by taking a series of simultaneous readings, the actual difference of elevation between the water surface

at the two places could be ascertained, from which an instructive table could be made out giving the available head for different depths of water flowing over the weir.

Useful Data and Formulæ Pertaining to Water.

One English gallon weighs 10 lbs. = 0·16 cu. ft.
One U.S. „ „ 8·34 lbs.
One cubic foot „ 62·5 lbs.
One cubic decimeter „ 1 kilogramme.
One litre „ 1 kilogramme.
One cubic metre „ 1 ton (metric).

If v = velocity in feet per second, and h = height of free fall in feet, then

$$v = \sqrt{2gh} = \sqrt{2 \times 32{\cdot}2 \times h} = 8\sqrt{h} \text{ approximately.}$$

If V = velocity in metres per second, and H = height of free fall in metres, then

$$V = \sqrt{2gH} = 4{\cdot}4\sqrt{H} \text{ approximately.}$$

A column of water one foot high exerts a pressure of 0·43 lbs. per square inch.

A column of water one metre high exerts a pressure of 0·10 kg. per square centimetre.

(One kilogramme per square centimetre = 14·2 lbs. per square inch.)

One atmosphere = 14·7 lbs. per square inch. (Note how closely this coincides with a pressure of one kilogramme per square centimetre. This is of great convenience in making approximate and rapid calculations in the metric system.)

For quantities of water per horse-power, see page 19.

CHAPTER III.

APPLICATION OF WATER POWER TO THE PROPULSION OF MACHINERY.

1. Historical sketch of the utilisation of water power and early types of wheels, with deductions concerning the power derived from them.

WATER power was applied to the propulsion of machinery more than two thousand years ago, and indeed there are evidences that the early Egyptians made use of the current in the Nile for corn grinding by contrivances the outlines of which have been engraved on enduring stone. The earliest exact record of a water-wheel applied to corn grinding is given by two authorities[1] as that of Antipater of Thessalonica in 85 B.C., and mention is also made by Strabo of a water-mill belonging to Mithridates King of Pontus. Later we find that there are evidences of watermills in Europe almost at the period of the Roman Justinian Code, and from this time their use for the purposes of corn grinding began to extend. Many of these early mills consisted of barges moored in rivers, from the sides of which paddle wheels on horizontal shafts projected, and these were slowly turned by the current as it swept past. Even to-day this type of mill is used in Southern Europe, though they are fast being replaced by more modern forms of mill. In England the use of water power was fairly extensive a thousand years ago, for we find surveys of thousands of

[1] "History of Corn Milling," by Richard Bennett and John Elton.

mills recorded in the Domesday Book, and the Norse mill was, it is stated, introduced into Ireland in the third century. This primitive mill consisted of a horizontal paddle-wheel mounted upon a vertical axis, to the upper end of which the mill stones were fixed, and this was employed instead of the quern, which had done service from remote times, and which was the only method employed for converting the grain into flour before the introduction of the watermill.

It is natural that the water-wheel should have been almost entirely associated with flour milling from the earliest times, for any new method of obtaining power would first be applied to the production of the staple food and afterwards to other purposes, though it is possible that the lifting of water for irrigation purposes may have been performed by some of the earliest and crudest water-wheels. We are not, however, as much concerned here with the applications of water power as with the way in which the power may be developed by the use of wheels, and the various types of wheel that have been evolved as the outcome of experience. It is sufficient to suppose that the necessity created by the demands for cheap flour led to the use of water power, and to the extinction of the quern in countries favoured by Nature with rivers which could be utilised for the purpose. The wheel thus introduced to the service of man has been applied to all manner of purposes, and water power was for many centuries the only supplement to animal power that was employed, unless windmills were introduced much earlier than existing records would have us believe. With the introduction of steam, water power has taken a very secondary place in those countries where coal can be procured at a

moderate price, but notwithstanding the competition of steam power, the vast improvements in the construction of water-power machinery and the utilisation of remote falls by electric transmission of energy, has led to a regeneration in the use of water power, of which we cannot yet foretell the ultimate influence upon the industries of those countries favoured with great powers in rivers and lakes.

It is very improbable that the reader would ever have occasion to design or construct a water-wheel of the type that preceded the introduction of the turbine, and therefore it is only for the sake of historical continuity that space ought to be given to descriptions of them, for they are fast becoming obsolete in England, and in many cases have been removed to make way for the more efficient machine. In one sense the extinction of the water-wheel is to be regretted, for of all the mechanical contrivances in or about a mill, they were the most picturesque, and the old wheel, groaning and creaking as it revolved between moss-covered walls, was by no means the least pleasing feature of an English landscape, and such may still be seen in places where the mill proprietor has not become imbued with the idea of making the most out of the water in his mill stream.

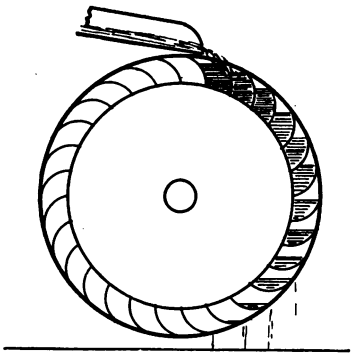


FIG. 8.—Overshot water wheel.

Three types of wheel (shown in Figs. 8, 9, and 10), have

been used for many centuries, while that shown in Fig. 11 is comparatively modern, and is called after the French inventor Poncelet. As the operation of this wheel involves principles which are employed in the modern turbine in their entirety, it forms in a sense a connecting link between the old wheel and the modern.

The oldest and most extensively used wheel in England was the overshot wheel, constructed of a series of buckets of sheet-iron or wood set round the periphery of the wheel and supplied with water from a shute or head race extending over the top of the wheel, so that the water entered the buckets approximately tangential to the circumference. The dead weight of the water in the buckets on the descending semi-circumference supplied the torque for turning the wheel. It will be seen from the illustration that a descending bucket lost a large part of the water it contained before reaching the lowest diameter of the wheel, so that the effective fall of the water acting upon the wheel was considerably less than the diameter; thus of a given fall of water, or difference of elevation between head and tail race, hardly more than half was utilised by this wheel. By shaping the buckets with care a greater amount of water could be retained than might appear from the illustration, which is purposely exaggerated to illustrate the loss which would inevitably occur in such wheels from this source. In order that the wheel should clear the water in the tail race it was designed so that the diameter would be slightly less than the difference between head and tail race. A further loss of available power was the result of this restriction. Wheels of this class have been constructed 80 feet in diameter, so that the head which could be utilised

would be about half this amount. This limitation prevented water wheels from being utilised to develop the larger water powers, and confined their application principally to small streams, upon the banks of which mills could be erected.

Rankine[1] states that the efficiency of wheels of this kind is as high as 70 to 80 per cent. when properly designed and constructed, but later experience of water wheels, combined with accurate tests upon turbines, throw discredit upon such a statement, and it is doubtful if such high efficiencies

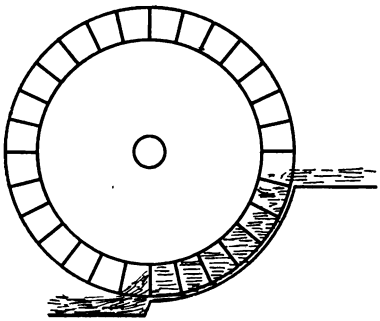


FIG. 9.—Breast water-wheel.

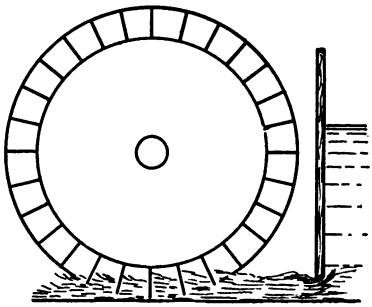


FIG. 10.—Undershot water-wheel.

were ever attained with this form of wheel. If such were the case, they would compare favourably with modern turbines for low heads, but only within a limited range. The expense of maintenance, especially if constructed of wood, is considerably more than that of a turbine plant of the same size, and the loss by friction owing to the necessarily heavy gearing is also greater with the water-wheel, as the machine work is less exact.

The breast wheel (Fig. 9) is so called in consequence of

[1] "Applied Mechanics," p. 628.

the arc of masonry or wooden sheathing that closely conforms to the periphery of the wheel from the point at which the water enters the wheel to the place of discharge, usually a little less than a quadrant. The water is confined in the wheel during the time of descent of the bucket by the breast, and acts upon the wheel chiefly by weight and to some extent by the velocity at which it leaves the head race. The maximum theoretical efficiency of these wheels may be as high as 80 per cent., notwithstanding the leakage of water between the wheel and breast, for close fitting was not a characteristic of the work of the millwright, as is shown by the old water-wheels. Rankine gives for the velocity of the periphery of wheels of the two foregoing classes 3 to 6 feet per second, which corresponds to 2·3 to 4·6 revolutions per minute for a 25 foot wheel, which may be taken as the average size of wheels used upon the mill streams.

The undershot wheel (Fig. 10) has radial paddles against which the water impinges when issuing at the velocity due to the head from beneath the sluice gate. The dead weight of the water does not operate upon the wheel at all, and the theoretical efficiency of these wheels is only 50 per cent., and their actual efficiency about 25 per cent. They were never much used in England, as a more efficient wheel of the breast or overshot type could be adapted in most cases. In Southern Europe they were used to a greater extent, in many instances being placed directly in the current of a stream or supported from a floating barge, and many such wheels were employed on the Rhine for corn milling, the efficiency of which was not more than 20 per cent.

The early forms of water-wheel were entirely constructed

of wood with the exception of the shaft, gudgeons, and the large gear for communicating the power to the machinery. The later wheels have cast-iron spokes secured to cast-iron bosses on a steel or iron shaft, and carrying at their outer ends the sole plate which formed the back of the buckets and the drum of the wheel. The buckets are made of sheet-iron in the best wheels, sometimes strengthened in the centre if the wheel is wide. The cast-iron toothed ring or gear, which meshes with a pinion, is bolted to one of the circular rims or "crowns" to which the spokes are fastened. Owing to the slow speed of the wheel, a high gear ratio is usually necessary between the wheel and mill shaft. The loss by friction in the gearing is heavy in consequence of the difficulty of keeping the gears in alignment, which prevents them from always meshing on their pitch diameters. This loss is considerably reduced in turbine transmissions, for the gears can be made with greater accuracy, sometimes cut gears being used instead of hard-wood teeth set into a cast-iron spider, which is a common mode of construction of the main gearing.

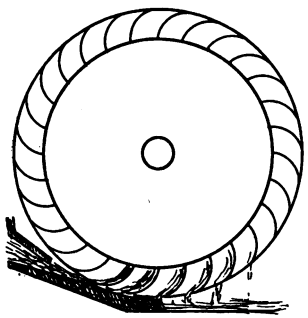


FIG. 11.—Poncelet water-wheel.

The Poncelet wheel (Fig. 11) receives the water at a velocity which is not quite that due to the total fall under which it works. The vanes are curved so that the water enters and exerts a pressure by being deflected, and subsequently it falls back along the vane and is discharged into the

tail race. The breast slopes away from the wheel so that the water may acquire velocity before entering the vanes. The diameter of the wheel does not depend upon the height of fall, and consequently these wheels may be constructed of any suitable size for a given fall. This type of wheel has been much used in France, and, according to Delaunay, it yields an efficiency of 56 to 60 per cent. under low falls, which is close to the maximum that has been reached by wheels of the types under review. The water leaves the wheel with a relatively low velocity owing to the curvature of the vanes, which impress upon it a velocity in a backward direction relative to the wheel. The water comes to rest at the top of the vane, and in descending exerts a further impulse in the direction of motion and flows away with a low absolute velocity, just sufficient to prevent backwater. A fraction of the head is lost in this wheel owing to the necessity of raising it above the water in the tail race so that it may revolve clear.

Actual tests for proving the efficiency of the water wheel have been made, but as the testing involves special appliances they are not usually convenient, and wheels of these types are rarely used for dynamo driving, except incidentally, such as for lighting the mill which is driven by the wheel. If exclusively used for electric work, an accurate estimate of the power developed may be read off from the ammeter and voltmeter when the efficiency of the dynamo is known. From tests of such a kind reliable inferences may be made if the water is measured correctly. It would appear from such information as exists, and it is but little, that water wheels fall off very rapidly in efficiency at partial gate as contrasted with turbines. This might be expected

from the heavy friction losses at full load, which are not sensibly diminished at partial gate, and therefore at partial gate there is a less proportion of the energy delivered to the mill shaft. With turbines the friction loss is less than in the mill-wheel, mainly, perhaps, owing to the cruder construction adopted by early millwrights.

CHAPTER IV.

THE HYDRAULIC TURBINE.

The modern turbine and conditions of utilisation.

THE turbine (which is superior to all other hydraulic motors) consists of a wheel with curved vanes or buckets through which water is allowed to flow. The wheel as constructed is in the form of a disc or cone, to which the vanes are attached at equidistant intervals round the circumference. The vanes may be disposed with reference to the axis in various ways, and may also be large or small compared with the diameter of the wheel. There is consequently a great variety of turbines, those at one end of the scale bearing but a slight resemblance to those at the other. Like most great inventions, the modern turbine is the product of the work of many men, for while certain forms of wheel are directly assigned to the skill of inventors whose names they bear, the successive improvements which have been added to the original ideas have resulted in perfected machines, which at the present time are the most efficient prime movers that the engineer possesses. There is no difficulty in returning 75 per cent. of the actual energy in water to a useful purpose by means of the water turbine. The steam, gas, or oil engine, on the other hand, compare very unfavourably in this respect. The most approved gas engine is incapable of yielding 40 per cent. of the actual energy supplied to it in the gas, while the steam engine

falls far short of this. In this connection it is expedient to refer to the steam turbine, if only by way of comparison with the water turbine. Though both machines embody the same essential principle of a series of vanes being set into motion by the movement of a stream of fluid across them, the comparison begins and ends with this statement, for the nature of the working fluid is so different in the two machines that the construction, principles involved, and results, bear but slight resemblance to one another. The hydraulic turbine utilises an incompressible[1] fluid, of comparatively great density, moving with a velocity which at the most is but a small fraction of the high steam velocity in the steam turbine. The volume of water issuing from the hydraulic turbine is the same as that at entry, but the expansion of the steam through the stages of a steam turbine results in a discharge a great many times that of the original volume. The high vane velocity again calls for a mechanical construction of great strength, but at the same time extreme lightness, so that the mass of rotating metal may not induce dangerous stresses. Moreover, the special problems introduced by expansion in the metal at the high temperatures in the steam turbine are not present in the water turbine, not only because the extremes of temperature are not so far apart, but also because the fitting is less accurate for hydraulic machines. The hydraulic turbine is, on the contrary, comparatively heavy, and the disadvantage of weight in the steam turbine may be turned to advantage in low speed hydraulic turbines by serving the purpose of a fly wheel. The leakage of steam through the annular space between the vanes and casing in a steam

[1] See page 16.

turbine is a serious item to be reckoned with in the construction of these machines, but leakage losses in water turbines are small, though the construction of the wheel and fit in the casing is of a much rougher description than that necessary in the steam machines. The expansion of steam necessitates a series of stages or rows of vanes between which there is a small difference of pressure. There is nothing of this kind in the water turbine, as generally

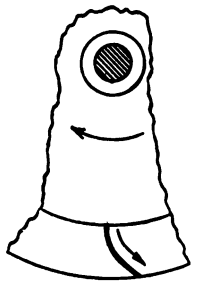


FIG. 12.—Radial outward flow turbine.

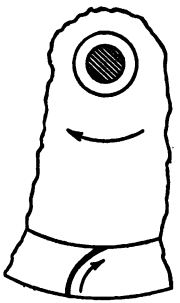


FIG. 13.—Radial inward flow turbine.

only one stage is employed, even in cases where the head of water is great.

VARIOUS TYPES OF TURBINE.

The pressure exerted by a stream of water upon a curved vane may be utilised in four ways to impart rotary motion to a shaft according to the direction of flow of the water relatively to the shaft. This gives rise to four types of turbine, designated respectively by the course which the water takes from entry to exit. They are as follows :—

1. Radial outward flow turbine (Fig. 12). In this

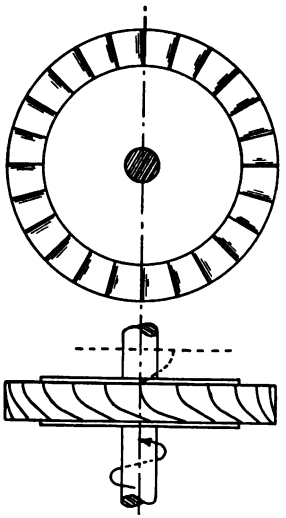


FIG. 14.—Parallel flow turbine.

wheel the water enters round the shaft and flows radially outward through the wheel and is discharged round the outer periphery.

2. Radial inward flow turbine (Fig. 13). In this wheel the water takes the opposite direction to that of No. 1. This is a more common type of wheel as the course of the water from outside to inside renders the construction simpler, for the guide vanes are thus outside the wheel and are easily accessible.

3. Parallel flow turbine (Fig. 14). The designation given to this type of wheel would indicate that the water enters and leaves the turbine parallel to the shaft. This is, however, not quite correct, for the course which the water actually takes is that of a spiral or helix. The course of the water in Nos. 1 and 2 is also spiral, but in a plane at right angles to the shaft.

4. The mixed flow turbine (Figs. 15 and 15A). This is a combination of 2 and 3. The water enters round the outer edge of the wheel, flows radially

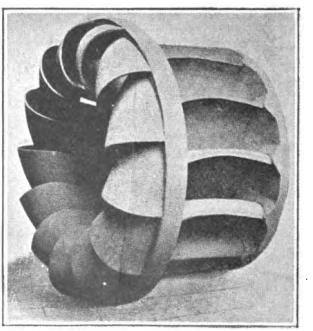


FIG. 15.—Mixed flow turbine.

inwards, and then in a direction similar to that of No. 3, where it acts upon the vanes which are bent round for the

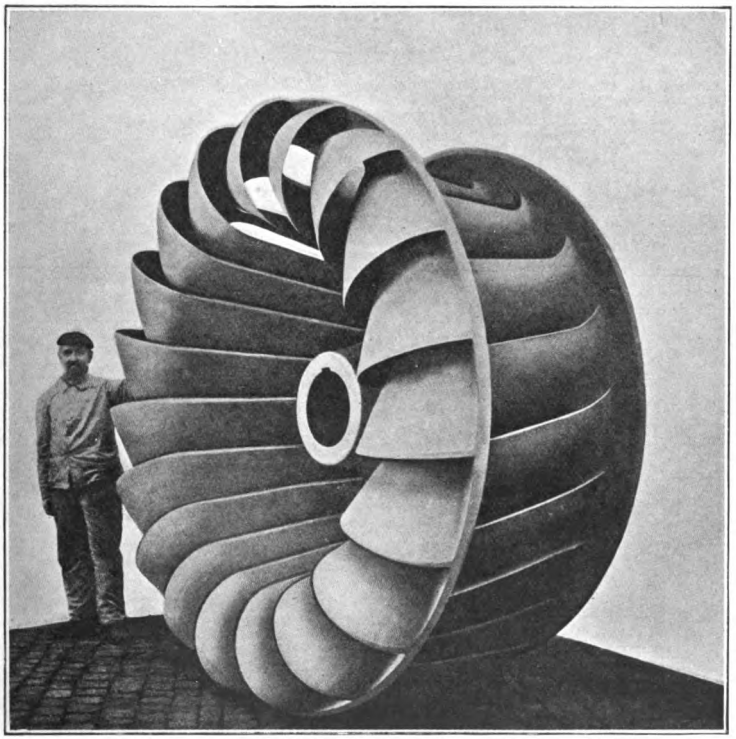


FIG. 15A.—Mixed flow turbine.

purpose of utilising the remaining kinetic energy in the fluid after it has passed through the radial vanes.

Any of these types of turbine may have horizontal or vertical shafts, according to the conditions under which they operate.

The foregoing classification of turbines would be incomplete without the necessary division of all turbines into two systems, into one of which every turbine must be placed. The distinction between these two systems is somewhat confused by a misleading nomenclature which does not convey the true sense of the difference between the two types. The terms "impulse" and "reaction" are not sufficiently explicit, but on the other hand it is difficult to replace them by a short definition which will convey the essentials without an additional explanation.

The subject is one which has received a new measure of importance since the steam turbine has entered the field as a prime mover, for the distinguishing features of the "impulse" or "reaction" water turbine are imitated in steam turbines of different types. As the words "impulse" and "reaction" have apparently come to stay we must perforce continue to make use of them until a more descriptive short definition be found. It will be remembered that pressure head and velocity head, corresponding respectively to potential and kinetic energy, are the two forms in which the energy stored in water may be utilised in operating machinery. Water under high pressure has been used in service systems for driving hydraulic motors essentially similar to slide valve engines. The motor consists of a piston working in a cylinder, and the water is admitted through valves alternately at each end, and is then discharged during the return stroke after it has accomplished the work of driving the piston forward against the resistance, whatever it may be. In such a case pressure head only is used, and the potential energy of the water is transformed into useful mechanical work upon

the piston. Supposing that, instead of admitting the water under pressure to the cylinder, the supply pipe be disconnected and the stream directed against a wheel with a series of buckets arranged round the periphery. The pressure head is now converted into velocity head, and the work done upon the wheel is due to the kinetic energy of the stream. These represent the two extremes of action in hydraulic motors. All turbines, however, operate under a combination of the two principles, and are designated as

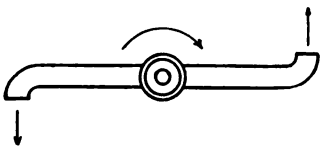


FIG. 16.—Tourniquet to illustrate the principle of reaction.

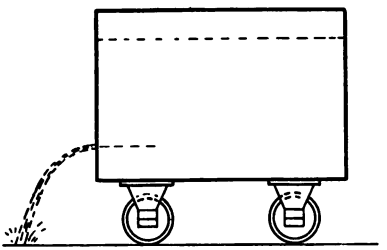


FIG. 17.—Tank on wheels, to illustrate the principle of reaction.

" reaction " on the one hand, and " impulse " or " action " on the other, according to whichever principle predominates.

The reaction caused by a stream of water is experimentally shown by Figs. 16 and 17. The first represents a tube with the ends turned at right angles in opposite directions. It is supplied with water through the pivot at the centre and it rotates in the direction of the arrow under the reaction of the issuing jets. This principle is directly applied to lawn sprinklers, for, by the rotation of the tube the water is scattered over a wider area than would be possible by means of a fixed nozzle. The tank supported on wheels,

from which water issues through a hole at the end, would acquire an acceleration backwards under the reaction of the jet. This force or reaction may also be accounted for by the unbalanced hydrostatic pressure, for the end of the tank which is not pierced has thus a greater area for the water to press upon. By closing the orifice the area on the two ends is equalised and there is no resultant pressure. Turbines are so designated which utilise this principle of reaction; but if the water is allowed to acquire the velocity due to the head before it enters the wheel, the turbine is working on the action or impulse principle and the pressure head is converted into velocity head. The pressure at the mouth of the nozzle through which water issues freely is almost nothing when the velocity is that due to the head, but when the water is confined in pipes or between the vanes of a reaction turbine, the pressure is maintained and the velocity is less than it would be if, as in an impulse turbine, the pressure did not exist. It is somewhat difficult to dissociate the two ideas of action and reaction in the abstract, but the difference between them is made clear by recognising in the impulse or action turbine a high velocity, and almost no hydrostatic pressure, and in the reaction turbine a low velocity with a pressure which is exerted against the containing walls or passages of the wheel, and which exists by reason of the fact that only part of the pressure head is converted into velocity head. The distinction between the two systems is also heightened by the fact that reaction turbines work "drowned," by being completely immersed in water. That means that the water entirely fills the passages, channels, and the spaces between

the curved vanes, and there is no air within the wheel. The impulse turbine, on the other hand, is located above the tail race, and the water which passes through it only occupies a part of the volume between the vanes, the rest being taken up by air at atmospheric pressure. In certain cases the entire space may be filled with flowing water, but the

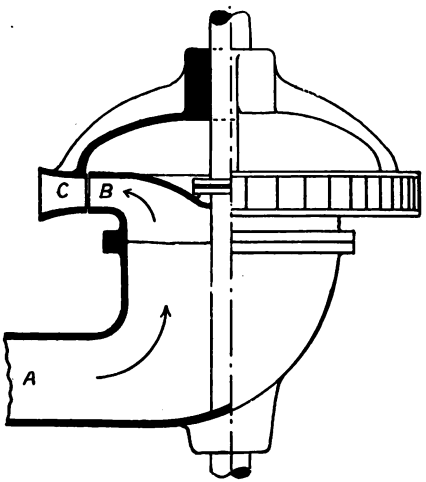


FIG. 18.—Fourneyron turbine.

discharge from the wheel is such that there is no back pressure exerted upon the vanes.

In all turbines, whether reaction or impulse, it is necessary to guide the water before entering the wheel so that it shall enter without impact and therefore loss of energy. This is accomplished by the guide passages or nozzles—the angles of which are generally fixed in direction, but in many turbines are arranged so that they may be turned about an axis by the governing arrangement, and thus the speed of the wheel can be controlled.

The French inventor Fourneyron (1802-1867) devised a turbine of the outward radial flow type to work as an impulse or reaction wheel, and his name has been universally applied to that particular form of wheel, still much used in France. Fig. 18, which is a sectional elevation of the Fourneyron wheel, illustrates the construction. The water enters through the penstock A, following the course of the arrow until it emerges from the guide vane B and enters the wheel C. The penstock may be either below the wheel, as in this turbine, or above it. Where the turbines work under high heads the arrangement shown is adopted for convenience. This wheel may be so proportioned as to operate upon either system, and there are guide vanes sometimes only round an arc of the circumference, in which case the turbine becomes a partial admission wheel, which also implies that it is operating by impulse alone. The shaft to which the turbine is keyed passes out through a watertight gland, and may be either horizontal or vertical as convenience suggests. The Fourneyron wheel is not extensively used outside the country of its origin, and chiefly for the reason that the regulation required for electrical work, for which purpose the greater number of turbines are built at the present time, is more difficult with the radial outward flow arrangement, for there is little room inside the wheel for the necessary mechanism for operating controlling gates, or for tilting the guide vanes as required by a change of load or water level. These turbines are usually regulated by a cylindrical gate interposed between the wheel and guide vanes, which alters, according to the position it is caused to occupy, the width of the passages admitting water to the wheel. The weight of this cylinder is balanced by a

counterpoise, so that the governing mechanism has only to overcome the inertia of the system when altering the flow of water to the wheel.

The Fourneyron turbine, though it may be worked as an impulse wheel, is usually "drowned," and therefore operates under the "reaction" principle. Impulse wheels as a class bear the name of another French inventor, and are known all over the world as Girard turbines, which take various forms and designs according to the conditions under which they revolve. They may be outward-radial or inward-radial flow wheels, and also parallel or axial flow, and yet be properly designated by the name of the engineer who made a series of early experiments upon turbines, and devised wheels suitable for high and low falls. They all employ the impulse principle of working, and the water is allowed to attain the full velocity due to the head before entering the wheel.

The illustration on page 66 shows a radial outward flow Girard turbine of 525 h.-p. for a fall of 95 ft. The external diameter of the wheel is 63 ins., and the water enters from below, and is discharged round the outer edge. The small horizontal shaft below the wheel proper is for the purpose of regulating, by adjusting the flow through the wheel. This turbine is now driving a textile mill, and the upright shaft, the lower part of which is seen in the illustration, is about 80 ft. long, at the upper end of which the power is taken off through bevel wheels on to a horizontal shaft. The weight of the vertical shaft and bevel wheel is partly borne by the hydrostatic pressure upon the wheel acting upwards, and it is for this purpose that the water is admitted below the wheel. This plan is frequently adopted

in large installations, and a very large part of the weight of the upper parts of the Niagara turbines are water-borne

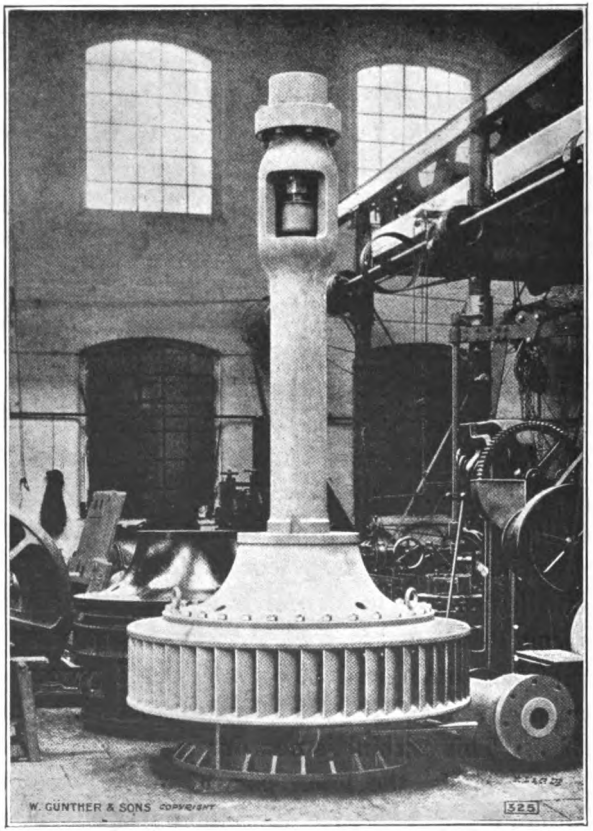


FIG. 19.—Radial outward flow Girard turbine of 525 h.-p.

in this manner, which relieves the step or thrust bearings of a heavy pressure which would render efficient lubrication difficult.

The reaction wheel extensively used in Europe for low falls is a parallel flow wheel which bears the name of Jonval.

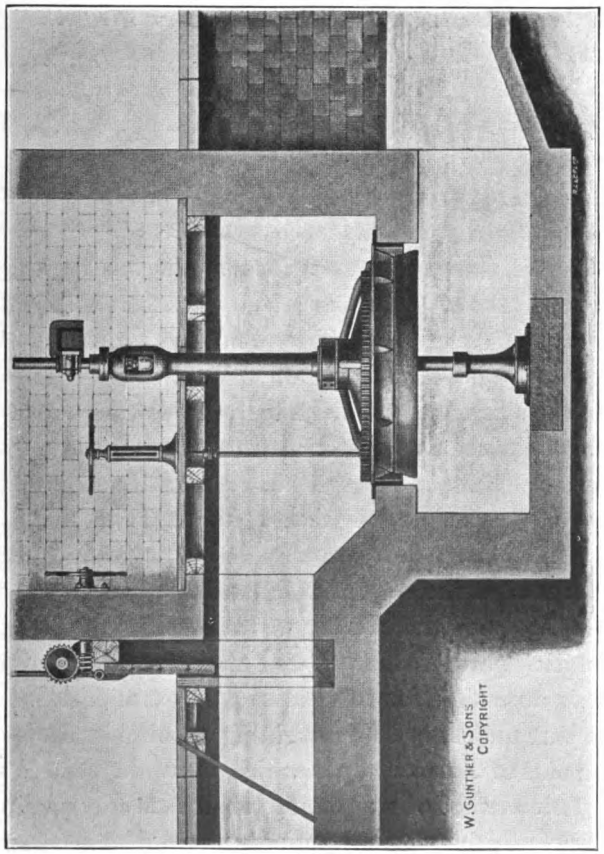


FIG. 20.—Jonval turbine in pit.

The water is, as usual, deflected by guide vanes, so that it enters the wheel making an angle with the shaft, and is discharged beneath into the tail race. These turbines

operate satisfactorily under very low heads. There is one such wheel giving 40 h.-p. under a head of less than two feet. The wheel is very narrow axially, but of very large diameter and passes a large volume of water. It is divided into two concentric sets of vanes, so that when the conditions in the river allow a higher fall, one set of vanes may be shut to the passage of water. The Jonval turbine generally does not give as high an efficiency as the Girard except under very low heads, but it gives very good results down to about one-half gate. Fig. 20 shows a Jonval turbine arranged to utilise a low fall of water. The control of the water to the wheel is effected through the vertical shaft and hand wheel, and the pinion at the bottom meshes with a large wheel, which, by being turned, regulates the size of the openings to the guide vanes. These vanes are contained in a cast-iron ring embedded in a concrete arch, which separates the head and tail races. The turbine proper is suspended from a hollow shaft through which a solid steel shaft passes. This latter shaft rests upon a pedestal at the bottom of the tail race, as shown in the illustration, and the bearing which sustains the weight of the rotating parts is thus placed well above the water line and is accessible for lubrication and examination. It would be best understood by imagining a thimble dropped over the point of a pencil, upon which it would be free to rotate. This method of supporting turbines is only possible for low falls, as otherwise the supporting solid shaft would be too long. Another method of supporting the weight of the turbine is shown in the illustration on page 137. In this case the bearing is also placed well above the water line, and is formed by a steel casting bolted to two heavy steel

joists, which sustain the weight. The bearing plates, which are keyed to the shaft, run in oil, so that only occasional attention need be given to them, and the turbine hangs from this bearing, and is guided in the centre by a light bearing placed close to the top of the casing. A large number of turbines have been erected with a bearing below water constructed of lignum vitæ, a hard wood which resists abrasion to a remarkable degree. It is, however, becoming an obsolete practice in turbine design to place the bearing which sustains the weight below water, and the advantages of a perfectly lubricated thrust bearing in an accessible position are now fully appreciated. With a lignum vitæ step bearing the water lubrication is sufficient, but with metallic rubbing surfaces an oil pump, with which oil is forced into a bearing below water, has been used, but with indifferent success.

The type of reaction wheel at present most in use is the mixed flow turbine. The illustration on page 59 shows this type of wheel as usually constructed. It will be seen that the course of the water is radially inwards, and then in its descent it encounters a series of vanes which are disposed in the same manner as in the Jonval turbine, and thus a double effect is produced. As actually constructed, the radial and parallel vanes are formed in one piece, the radial vane being turned up at the end to form the parallel vane. They are usually made of sheet steel cast into the wheel, but are sometimes cast with the wheel, though in the latter case the two sets of vanes are constructed separately, and the rings are turned, faced true, and bolted together. The best vanes, however, are those of sheet steel, as the smooth surface diminishes the loss by

skin friction, and the sharp edges also tend to make the flow through the wheel more regular.

This type of wheel will work under low and moderate falls with very high efficiency, and it is only under very high falls that its utility is contested by the impulse wheel. The diameter of these wheels for a given power is relatively small, but they are often made long in the direction of the shaft, so that they pass a large quantity of water. As all the water entering the wheel has to flow parallel with the shaft in the annular space within the wheel, the length shaftwise is limited for this reason. The shaft may be horizontal or vertical, and for high heads the horizontal position is frequently adopted, two wheels being placed together upon a single shaft. The design of the mixed flow turbine to develop a given horse-power under a specified head is a problem which can only be solved by the light of practical experience, rather than by rules and calculations. The manufacturers of turbines ordinarily require the following data from which to construct a wheel.

(1) Available head under which the wheel is to work, and consequently the velocity of free fall of the water due to the head.

(2) The variation in the head due to the possible backing up of the water in the tail race during floods.

(3) The quantity of water which is available at all seasons of the year in litres or cubic feet per second. If the flow is very variable, and it is desired to utilise as much of it as possible at all seasons, it may be necessary to install more than one turbine, the capacity of the units being such that the flow during the dry weather will be sufficient to keep one of them in operation at full load, and consequently at

CLASSIFICATION OF TURBINES.

Reaction Turbines.		Impulse (or Action) Turbines.
Distinguishing characteristic:—Wheels of this class work "drowned," with the water under pressure in the wheel, and all parts full.		Distinguishing characteristic:—The water enters the wheel with the velocity due to the head. The spaces between the vanes are only partially filled with water, and air is allowed to penetrate to all parts of the wheel. Generic name:—Girard.
Parallel Flow . .	Jonval (invented 1841).—Especially applicable to very low and variable falls, with variable volume of water.	Fontaine.—These wheels may or may not be immersed. They are often arranged to work by partial injection, the water acting upon a fraction of the whole number of vanes.
Radial Flow . .	(1) Outward. — Fourneyron (invented 1823).—The difficulties of controlling this wheel and the cost of construction in comparison with other types have caused it to be superseded. (2) Inward. — Francis. — Sometimes called Vortex according to its construction. (3) Mixed.—The most widely used principle in the modern turbine. It is a combination of the inward and parallel flow types. Generally called Francis.	(1) Outward.—Fourneyron.—These wheels are not immersed, and operate by partial injection.
Tangential Wheels	None.	"Pelton" or Jet Wheel. — This is the most satisfactory turbine for exceptionally high falls. Though not as efficient as wheels operating under medium falls, they are the most efficient for exceptionally high water velocities.

maximum efficiency. Sometimes the flow is superabundant, in which case the horse-power actually required throughout the year must be known.

(4) The speed of the shaft from which the machinery is to be driven, and the kind of machinery that it is required to drive. The direction of rotation of the shaft is important for direct drives, but with gearing between the turbine and line shaft the required direction of rotation can usually be obtained by a suitable arrangement of the gears, and is thus independent of the direction of rotation of the wheel. The makers of wheels usually supply them to run in the direction of the hands of a clock (unless otherwise specified) for wheels with vertical shafts where the observer is placed above the wheel.

The table on page 71 shows the variety of wheels with the names by which they are usually designated. Besides the general distinctive names by which turbines are classed, there are a host of " brands " favoured by the makers, which have generally some feature upon which they are wont to place great reliance, but which seldom sustain the often extravagant claims made for them by the would-be vendors.

The writer has been assured of 95 per cent efficiency by a turbine maker, who doubtless considered himself safe from a disclosure of the real efficiency of his wheel by the fact that the process of carrying out tests on turbines to ascertain the exact facts is generally costly and seldom resorted to. In this respect the purchasers of turbines are entirely in the hands of an unscrupulous maker, and where a lack of technical knowledge exists on the part of the purchaser, he is likely to be governed by every other consideration than the right one in purchasing a

wheel, and, be it said, there is only one suitable type and size of wheel for each particular case. To ascertain this correct type and size a knowledge of the site and purposes for which the wheel is desired are necessary, together with the discrimination of the experienced engineer. Many makers of turbines have their standard sizes and types, from which a selection may be made by the engineer of the purchaser, and often the result is quite as satisfactory as if a wheel were specially designed to fit the case in hand. It costs considerably more in the foundry and machine-shop to turn out a special wheel for which working drawings and patterns have to be made; but this extra cost is sometimes completely justified by better adaptation, resulting in higher efficiency. For places where water is scarce, an extra 5 per cent. efficiency may mean the success of the plant, to secure which a slight additional first cost may be justly entailed. It is chiefly for small isolated plants that wheels are sold without adaptation to the requirements, for larger installations are designed with attention to details and usually under skilled technical advice. The large Italian and Swiss installations are perfect examples of design generally, in which every part of the machinery is especially adapted to its duty and is cleverly executed.

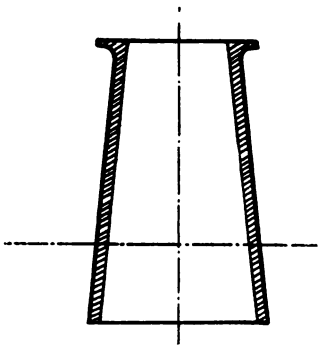


FIG. 21.—Cast iron draft tube.

It is sometimes convenient to raise a turbine above the level of the tail water for a considerable height, and, to allow

this to be accomplished and still work the wheel with all the parts full of water, draft tubes are used. These are either of cast iron (Fig. 21) or riveted steel, and are fastened beneath the turbine, and the discharge from the wheel passes through the tube into the tail race. The mouth of the tube dips beneath the surface of the water in the tail race so that air cannot enter, and as all the joints are made air-tight, both turbine and tube are entirely filled with water while working. The effect of excluding the air is to create a suction, which exactly compensates for the raising of the turbine above the tail water, and thus there is no loss of head. Draft tubes are frequently referred to as suction tubes for this reason, and a turbine may, therefore, be located at any elevation above the tail water up to a height theoretically equal to that of a column of water which would be supported by atmospheric pressure;[1] but practically, owing to leakage of air, suction is not efficient above 20 ft., which is the limit to the height above the tail water that the wheel may be placed and still utilise the full head. Generally the draft tube is much shorter than this, and is just sufficient to raise the turbine to a convenient level above the water, allowing for a possible rise in the level during floods. In the plant illustrated on page 67 a suction tube was unnecessary, as the normal tail water level is above the floor of the turbine pit and air cannot enter beneath, so that the turbine always works drowned. Draft tubes from 6 to 10 ft. in length are common in turbines of such a design.

[1] A water barometer would have a height, were a perfect vacuum possible, of $\frac{14{\cdot}7 \times 144}{62{\cdot}5} = 33{\cdot}9$ ft., corresponding to a mercurial barometer of 29·92 ins.

Fig. 29, on page 99, shows a forked draft tube fitted to a Niagara turbine of 5,500 h.-p.

Principles under which Turbines Operate.

It has already been pointed out that the stored energy in a given mass of moving water is proportional to the square of the velocity, or to the height through which it has fallen, unless during the descent there be frictional resistances present, to overcome which part of the energy has been dissipated. But assuming that, instead of a mass of water, there be a continuous stream issuing as a jet from a circular orifice with a velocity v, it is desired to ascertain how we may apply the power in the best manner, and without undue loss, to the propulsion of a water wheel. If A be the sectional area of the jet, the weight of water passing per second will be $A\,\gamma\,v$ and the mass M will therefore be $A\,\gamma\,v/g$. The power of the jet will therefore be

$$\frac{1}{2}\,M\,v^2 = \frac{A\,\gamma}{2\,g} \times v^3.$$

This expression appears to contradict the above assertion, as the square of the velocity is replaced by the cube, but it is easily seen that the weight of water passing a given section of the jet per second is proportional to the velocity, and that the above expression represents, according to the units chosen, the kilogramme-metres or foot-pounds which the water can yield up each second. If this quantity be divided by the horse-power constant (75 kg.-m. or 550 ft.-lbs.) the horse-power of the jet is ascertained. The simplest but at the same time the most wasteful way by which the power of the jet could be utilised would be by the

insertion of a pressure board as shown in Fig. 22. The pressure upon the board might then be utilised to drive it forward, and by attaching a series of these boards to a wheel in the manner of a paddle wheel, a part of the power might be successfully reclaimed—but only a very small fraction of the available supply.

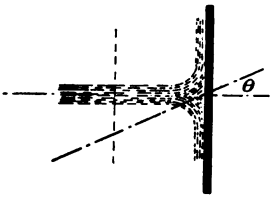


FIG. 22.—Jet impinging on plane surface.

First as to the pressure upon the board caused by the impact of the water. The mass of water which strikes the board each second is $\frac{A \gamma v}{g}$, and the momentum of this mass is $\frac{A \gamma v^2}{g}$. The pressure caused by destroying this momentum per second is therefore

$$P = \frac{A \gamma}{g} v^2.$$

The board is assumed to be held stationary in the stream, and the energy of the water is therefore expended in useless splashing, part of it being carried away with the fluid as it spreads out on the board in all directions. The efficiency therefore is zero when the board is held fast. Assuming that the stream has a sectional area of 4·2 sq. ins. and that the velocity, corresponding to a head of 18 ft., is 34 ft. per second, the pressure upon a stationary board would be

$$P = \frac{4{\cdot}2}{144} \times \frac{62{\cdot}5}{32{\cdot}2} \times (34)^2 = 65{\cdot}4 \text{ lbs.}$$

In order to utilise this pressure the board must move in the direction of the stream. This movement, at a velocity

which will be designated by V, causes a reduction in the velocity with which the water impinges, and this impinging velocity is $v-V$. The weight of water striking the board per second is no longer proportional to v but to $v-V$. The pressure acting on the moving board is therefore

$$P = \frac{A\gamma}{g}(v - V)^2$$

and the work done on the board would be

$$PV = \frac{A\gamma}{g}(v - V)^2 V.$$

This equation shows, first of all, what is manifestly true, that when the board is stationary, and V consequently nothing, the power is nothing. It now remains to ascertain what the speed of the board should be relative to that of the water so as to obtain a maximum power from the stream. This value of V is found to be $\frac{1}{3}v$ by differentiating the above equation[1], so that the maximum power which, under any circumstances, can be obtained by the board is attained at a velocity which is one-third that of the water. By substituting $v/3$ for V, and ascertaining the ratio of the power reclaimed to that imparted by the stream, the efficiency is disclosed.

$$\text{Efficiency} = \frac{\frac{A\gamma}{g}\left(v - \frac{v}{3}\right)^2 \frac{v}{3}}{\frac{A\gamma}{2g} \times v^3} = \frac{8}{27} = 29{\cdot}6 \text{ per cent.}$$

If there be a succession of boards, as in an undershot water wheel, the work done would be

$$PV = \frac{A\gamma}{g} v\, V\,(v - V).$$

[1] See Appendix A.

This would be a maximum when $V = v/2$ corresponding to an efficiency of 50 per cent. (see page 51).

Even if there were no losses in the machinery through which the power was conveyed to the point of useful application, less than 50 per cent. of the energy in the water could be usefully applied by this means. When the frictional losses are taken into account the efficiency drops to such a low figure as to render such water motors unworthy of the serious attention of engineers. It is not possible to apply pressure boards in practice in this manner, for if attached to a wheel the angle of inclination to the stream would be continually changing, and only at one point in their course would they be normal to the direction of flow. Moreover, their velocity in the direction of flow would be continually changing also, owing to the arc swept through by the wheel while the board is acted upon by the jet. The undershot wheel described in the last chapter is the nearest practical application of this principle.

The most perfect form of apparatus for utilising the kinetic energy of moving water would be that from which the water leaves the pressure boards, vanes, or buckets, with minimum velocity, carrying away a small proportion of the original energy. Clearly, there must be some residual velocity so that the water, after having passed through the apparatus, may be discharged into the tail race. It is the aim of the turbine designer to keep the velocity of discharge as low as possible consistent with a free flow away from the wheel. The successful attainment of this desired end is not to be found in devices which employ the force of impact of moving water upon a plane surface, for, as we have seen, some of the energy is expended in breaking

up the stream, and the rest is carried away by the high velocity of discharge along the face of the plane radially outwards from the point of impact. The effect of inclining the plane to the direction of flow is to reduce the normal pressure on the plane to $P \sin \theta$, where θ is the angle made by the plane with the direction of flow.

The principle upon which the water turbine operates was foreshadowed in the Poncelet wheel, which is designed with curved buckets over which the stream of water is intended to flow. In this respect it constitutes a notable exception to other forms of water wheel, which were caused to rotate by the preponderating effect of a full bucket on one side, against empty ones on the other, in the same manner that two cars upon a funicular railway, at the ends of a rope passing over a pulley at the top of the incline, are moved up and down by alternately filling the water tanks beneath the cars, thus providing a force for accelerating the mass and overcoming friction. In such a case the potential energy of the water, as distinct from the kinetic, is alone employed, and the parallel might be extended to the utilisation of the kinetic energy by imagining that the water, stored at the top of the incline, was allowed to descend in a stream and thus acquire kinetic energy to be used in this form to drive machinery to operate the cars. Now the turbine essentially depends upon the movement of water over curved vanes, and not upon buckets filled with stagnant water which, by their dead weight acting at the end of a lever, apply a turning moment to a shaft. The first principle has now entirely superseded the latter, and a much greater efficiency in hydraulic motors is the result of the change; thus, by passing from a consideration of the

older type of water wheel to the modern turbine, we pass from one principle to another.

If a stream of water be flowing along a flat surface A B (Fig. 23), in the direction of the arrow, a slight but inconsiderable force is exerted upon the plane tending to carry it in the same direction as the water, due to the friction of the water upon the surface. But as the plane is supposed to be very smooth, this force will be so small as to be negligible. If the plane, originally straight from C to B, be turned up about a hinge at C so as to occupy the position C B_1, and the water is still flowing, a force is developed by the change in the direction of the water at C, and this

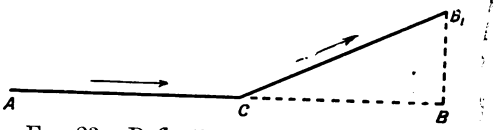


FIG. 23.—Deflection of stream of water.

force may be resolved into two components, one at right angles to C B and the other in the direction of C B. It is the first of these components that is made use of in turbines, to act tangentially to the wheel and thus cause rotation. In general, therefore, it may be said that the force employed in turbines is that due to the resistance which a mass of moving water offers to an enforced change in the direction of motion, and this change in direction is obtained by the contour of the surface over which the water is flowing. If, instead of a plane surface as shown in the figure, the surface be curved, the direction of flow of the water in passing over it is constantly changing, and this results in a pressure upon the surface at every

point. The water, during its passage through or over the moving vane, is constantly yielding up its kinetic energy, and if all the kinetic energy were given up it would come to rest in the vane, which of course is not possible in practice. There must therefore be a residual velocity sufficient to allow the water to escape, and the energy that is lost is therefore proportional to the square of this residual velocity. It is the aim of turbine designers to keep the loss from residual velocity as low as possible, and in practice it may be brought down to less than 6 per cent. of the theoretical velocity due to the head of water acting on the wheel. For instance, for a turbine working under a head of 22·8 ft., corresponding to a velocity of free fall of about 38·3 ft. per second, the water leaving the wheel flows away at a speed of about 9·2 ft. per second. The proportion of the initial energy passing off in the discharge is therefore $9{\cdot}2^2/38{\cdot}3^2 = 5{\cdot}8$ per cent.

The force which a mass of water moving with a velocity v in a given direction exerts against a compulsory change in the direction of its motion varies with the degree of change of direction imposed upon it, and increases with the angle through which the stream is diverted. This force may be simply illustrated by the tendency of a free hose pipe to straighten when water is first admitted to it. Other examples, equally simple, will doubtless suggest themselves to the reader. Another way of stating the same fact would be that the water has an acceleration imparted to it in a direction other than the course of the stream, and the force necessary to effect this acceleration is the pressure between the moving water and the walls of the containing vessel, or of

the curved vane along the surface of which the liquid flows. This is precisely the same force which causes fly-wheels to burst unless they are properly designed, for each piece of the rim of the wheel is constantly compelled to change the direction of its motion in space as it rotates about the axle, and the effects of this force have to be guarded against by the skill of the designer in proportioning the parts of the wheel in such a manner that the stresses induced will not overcome the tensile strength of the material. As the phenomena brought about by consideration of the action of centrifugal force are familiar in principle, the forces utilised in the water turbine may, at the outset, be more easily explained by dealing with the water flowing along the curved vane and exerting a pressure thereon in the same manner that the centrifugal force of a rotating body may be computed. In the simplest case, that of a vane conforming to the arc of a circle, and charged with a stream of water moving at a velocity v and entering and leaving the vane tangentially, we have an exact parallel to the ordinary problems in centrifugal force familiar to the engineer. In Fig. 24 the stream, with a cross-section A, is shown following the course of the vane, and discharging at right angles to the direction of entry. Even with the smoothest vane possible and the most careful adjustment of the nozzle it would be impossible to avoid considerable friction loss, with a consequent reduction

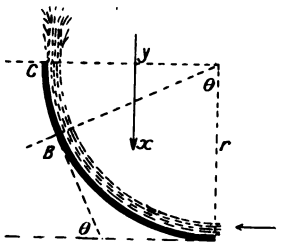


FIG. 24.—Pressure exerted on curved vane by stream of water.

of velocity, together with eddies and whirls which would mean a loss of energy. But for the moment these will be assumed to be non-existent, and consequently the velocity of discharge will be the same as that at the point of entry. If, as before, γ be the unit weight of water (unity in metric units or 62·5 in British) the weight of water entering the vane per second will be $A\ \gamma\ v$. At any point B on the vane the weight of water over a unit of vane area will be $\gamma\ A$, and, as v is the velocity of the water, the centrifugal force, at a radius r, or the pressure upon the vane at that point will be

$$p = \frac{\gamma\ A}{g}\ \frac{v^2}{r}.$$

This same reasoning applies to all points of the vane, so that it will only be necessary to integrate all these pressures, which act at all points normal to the surface of the vane, to obtain the resultant or total pressure in magnitude and direction. If it is desired to obtain the total pressure in the direction $x\ y$ normal to the initial direction of flow, consider the total pressure exerted upon the semi-circumference of a cylindrical boiler in which the unit pressure is p. This is easily shown to be $2\ pr$, or the total pressure upon a quadrant is pr. The total pressure therefore exerted by the water would be

$$P = pr = \frac{\gamma\ A}{g}\ \frac{v^2}{r} \times r = \frac{\gamma\ A}{g} \times r^2.$$

As the quadrant shape for the vane is symmetrical, the total pressure acting at right angles, or in the direction of the inflow, is also the same. It will be observed that this pressure exerted upon the curved vane is the same as that calculated for the pressure board placed normal to the direction of flow. In either case the stream is diverted through

an angle of 90°, and flows off with undiminished velocity and with the same energy as it originally had, except such as would be lost by splashing in the one case and eddies or whirls in the other. If the stream be diverted through an angle θ (less than 90°) which would happen if the vane terminated at B instead of C (Fig. 24), the pressure upon the vane in the direction of flow at entry would be obtained by taking the difference in momentum per second between the incoming and outgoing stream in the required direction. The pressure would then be

$$P = \frac{\gamma A v}{g} \times v - \frac{\gamma A v}{g} (v \cos\theta) = \frac{\gamma A}{g} v^2 (1 - \cos \theta).$$

Thus if θ be 57° and the conditions the same as in the example given for the flat board, the pressure P would be

$$P = 65{\cdot}4 (1 - \cos 57°) = 65{\cdot}4 (1 - 0{\cdot}54) = 30 \text{ lbs.}$$

As the vane is assumed to be stationary and the water leaves without loss of energy, it is clear that no work is done. When, however, the vane is in motion, work is done by the water. This movement is supplied by the rotation of the wheel under the action of the force, and the product of the force and the speed of the wheel give a measure of the useful work accomplished.

Without attempting to discuss the intricate questions involved in the design of turbines, as regards the correct angles and shapes for the vanes under various conditions of service, it will nevertheless be expedient to point out the nature of the problems which the turbine designer has before him. They are vastly complicated by practical questions which render abstract theory difficult, if not impossible, in many cases, to apply with certainty, but

there are applications of theory which have become indispensable in this connection, and, with their limitations known to the skilled engineer, they become useful tools.

The pressure of water upon a curved vane has been deduced on the assumption that the vane was stationary. But if the vane be put into motion the pressure is no longer the same. The velocity with which the water impinges is reduced if the vane has any motion in the direction of the impinging stream, and consequently the pressure falls. At the same time the absolute velocity of the water also falls, and therefore part of the original kinetic energy is transferred into work done against the resistances to the motion of the vane, *i.e.*, to drive the machinery to which the turbine is geared. To keep the losses as low as possible in a turbine, the water must enter the spaces between the vanes without shock. To ensure that the loss occurring from this source shall be small, the stream must meet the vane tangentially so that there be no impact. This in itself is a difficult condition to comply with owing to the variable speed of the wheel, so that, should the angle of injection be correct for one speed, it would not be so for another. In practice it is usual to adjust the angle so that at full gate the turbine may be running with maximum efficiency.

In Fig. 25 the edge of a moving vane AB is shown, the motion being in the direction of the arrow, and the water enters in a direction along the guide vane CD. It is then deflected by the contour of the surface of the moving vane AB, and is finally discharged at the other end, having sustained a loss in kinetic energy by the work done in propelling the vane forward against resistances. Supposing

that the velocity of the stream in the guides be v, it is clear that if the vane were held stationary the correct angle of incidence for the stream would be HAG, so that the water would enter the vane tangentially. If, however, the vane acquires a uniform velocity V, this angle must be altered so that the water shall still enter parallel to the edge of the vane, and thereby no energy shall be lost by impact except that due to the unavoidable parting of the stream by the edge of the vane. The water before it enters the wheel has

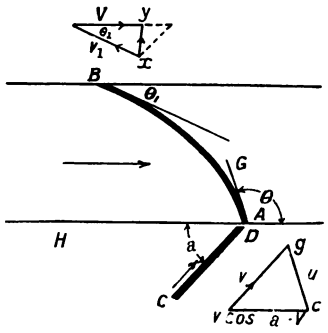


FIG. 25.—Moving vane of reaction turbine.

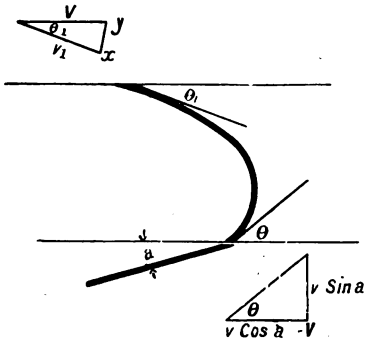


FIG. 26.—Moving vane of impulse turbine.

a velocity v due to the head. It is important to observe that this velocity is taken relatively to a fixed point and is therefore *absolute*. The component of this *absolute* velocity in the direction of motion of the vane is $v \cos a$, and as the vane is itself moving in the same direction with a velocity V, the *relative* motion of stream and vane is $v \cos a - V$.

Moreover the force acting upon the vane depends upon the difference between the velocity of the stream and that of the vane. The injection of the water without

shock can only be effected by adjusting the angles to the ratio between v and V, both of which are subject in practice to independent variation. The speed of the wheel, unless under the control of a governor of exceptional excellence, will vary with the load, and in small hydraulic installations a variable head will cause fluctuation in the initial velocity. The angle α, though unalterable in many turbines, in some designs is capable of being slightly changed through mechanism operated by the governor. The result of this is to alter the flow to the wheel as well as to change the incident angle. This method of governing is fully dealt with in another place.

The fixed or normal angle for the guide vanes is in all cases that which best satisfies the velocity ratio for the speed at which the wheel is designed to run. Clearly the angle θ which the wheel vane makes with the direction of motion must be of such a magnitude that, with the wheel at the normal speed, the water shall enter tangentially, but as θ and α are connected together by a definite relationship, a variation in the one renders a new value for the other necessary if the condition of entry without shock is to be adhered to. It is of course impossible in turbines to vary the angle θ. The wheel is designed with the vanes permanently fixed at an angle with the tangent which the conditions of service enjoin. The vanes, either of cast iron or steel plate, in the one case cast with the wheel, in the other cast into the wheel, are not capable of alteration after the wheel is set to work. Any alteration therefore in the angle α is not accompanied by a corresponding change in θ, and there is consequently a loss of energy at the entry owing to the imperfect guidance of the jet into the

wheel. As $v \cos a - V$ is the component of the relative velocity of the stream to the vane in the direction of motion, and $v \sin a$ is the component at right angles, by dividing the one by the other the tangent of the angle θ is obtained,

$$\tan \theta = \frac{v \sin a}{v \cos a - V} \tag{1}$$

$$\sin \theta \, (v \cos a - V) = v \cos \theta \sin a \quad \text{(2) or}$$

which, expressed in another form, is

$$\frac{V}{v} = \frac{\sin (\theta - a)}{\sin \theta} .$$

The angle θ varies in practice from about 150° to 0°. With reaction turbines it is generally somewhat greater than 90°, but many wheels are designed with the vane edge at right angles, though 95° is a common value. It should be remembered that the angle is here measured from the line of motion. Some writers use the line at right angles to measure from, in which case the common value as given above would be 5°. It is, however, more convenient to refer to the angle as measured from the line of motion. For impulse wheels the angle is often much less than 90° (Fig. 26), and for the impulse tangential wheel known as the Pelton it is 0°. For each value of θ there is, according to the above equation, a corresponding value of a which may be calculated when the ratio V/v is known. It can be proved from initial considerations that the best velocity for the wheel is one-half that of the water, but in practice, owing to friction, and the necessity for keeping the efficiency high over a considerable range in speed, the actual wheel velocity varies considerably from the theoretical, especially

with reaction turbines, and, though frequently less than one-half that due to the head, is also often higher than that which theory points to as the most desirable. Mr. G. R. Bodmer[1] gives a list of turbines in which the actual speed of the wheel is compared with the velocity due to the head as derived by experiment. Of 18 wheels operating under heads which vary from 5·1 to 18·1 ft., this ratio lies between 0·461 and 0·770. Of 14 other wheels made by a single Swiss firm, and operating under heads varying from 3·2 ft. to 1804·5 ft., the ratio varies from 0·377 to 0·68. Of these wheels, one has a speed which is exactly half that due to the head, seven revolve faster, and six slower, than the best speed according to theory, though at these speeds they are presumably yielding a maximum efficiency.

Taking 0·77 for example, as the ratio between V and v, and assuming that $\theta = 90°$, the value of a would be obtained by substitution in the foregoing equation or

$$0{\cdot}77 = \frac{\sin (90° - a)}{\sin 90°} = \cos a \quad \therefore a = 39°.$$

With $\theta = 29°$ and $V/v = 0{\cdot}48$ we have

$$0{\cdot}48 = \frac{\sin (29° - a)}{\sin 29°} \quad \therefore \sin (29° - a) = 0{\cdot}48 \times 0{\cdot}48$$

$$\sin (29° - a) = 0{\cdot}2304.$$

Therefore $a = 15° 42'$.

This is a value attained in impulse turbines, v being the velocity due to the head. For reaction turbines the entering velocity is less than that due to the head. If $\theta = 0$, as in the case of the Pelton wheel, it will be found by substituting in the equation (2) that $a = 0$, and this

[1] "Hydraulic Motors, Turbines, and Pressure Engines," G. R. Bodmer.

value is independent of the ratio of V to r, showing that whatever speed the wheel may run at the angle of injection is always the same for conditions of no loss of head by shock. This is self-evident from the tangential relationship of the wheel and jet. It is therefore the only form of wheel in which, for varying speeds, the condition of no loss by shock at the vane is observed, and were it not that it has other drawbacks it would, by avoiding a loss inevitable to other types of turbines with varying speeds, have an advantage over them in efficiency.

The angle at which the water enters the wheel obviously cannot be the same throughout the cross section of the stream, as the cross section diminishes towards the wheel in radial turbines in consequence of the reduced diameter of the circle as the wheel vanes are approached. The vane angles are therefore at best a compromise, and as a consequence there is always in practice a loss at injection consequent upon the impossibility of constructing a wheel which shall fulfil the above conditions in all respects. The calculated angle is that which would be correct for a very thin stream of water approaching the wheel, nevertheless in fixed vane turbines the angles are laid down within 5 or 6 per cent. of the calculated angle as found by assuming a ratio of velocities.

The water having passed over the vane is discharged at a velocity *relative to the vane* which will be called v_1 (Figs. 25 and 26). The value of v_1 would be nearly that of the velocity of the water *relative to the vane* at entry (which is $c\,g$ on the diagram or u), the difference being due to the energy lost in overcoming frictional resistances and in internal friction of the particles of water upon each other, as shown by the

formation of eddies and whirls. If the wheel be held stationary, $u - v_1$ is small, and v_1 becomes the absolute velocity of the water measured in the same manner as v—*i.e.*, *relative to a fixed point*, and as the vane is fixed it will be convenient to refer to v_1 as the velocity at exit *relative to the vane*. Now let the vane be put into motion by the force which the water exerts upon it owing to the enforced change from its original direction, and immediately work is being done upon the wheel, and the water consequently must lose kinetic energy and therefore must suffer a reduction in the *absolute* velocity with which it issues forth from the vane. Referring to Fig. 25, the *absolute* velocity of the water may be graphically obtained by the simple geometric construction shown. If θ_1 be the angle that the edge of the vane makes with the direction of motion of the vane, and V and v_1 be laid off to a convenient scale, and parallel to the respective directions of motion, the third side of the triangle formed by these two lines will represent in magnitude and direction the *absolute* velocity of the water. This line is $x\ y$ in the figure. Assuming for the moment that $u = v_1$, which means that the friction is nothing in the vanes, it may be seen by an inspection of the triangle that, whatever the angle θ_1 may be, the absolute velocity as represented by the length of the line $x\ y$ would become equal to v when $V = 0$, that is, when the vane is fixed. As V is reduced the line $x\ y$ approaches v in length until the area of the triangle becomes nothing and v_1 and $x\ y$ are coincident. In the practical design of turbines the angle θ_1 is so chosen that, with the assumed values of v_1 and V, the water shall leave the wheel as nearly as possible at right angles to the direction of motion. As

will be evident from the designs of turbines, this direction is approximately radial in the case of radial turbines and axial with parallel flow wheels. The exact attainment of this object renders the loss due to the energy escaping with the water a minimum, for the line $x\ y$ is shortest when at right angles to the direction of flow, considering V as the variable and θ_1 and v_1 as constants.

The losses of energy which are inevitable in the utilisation of water power are divided between the turbine proper, the gates and controlling apparatus, and the pipe, penstock, or flume, which conducts the water to the wheel, together with that carried away in the discharge. For the present we shall alone consider the losses in the turbine and discharge, so that the efficiency of the wheel is represented by the ratio of the power developed in the shaft of the turbine to that entering with the water into the guide passages. If P be the foot-pounds of energy per second exerted at the shaft, and W the weight of water in pounds entering the wheel per second, E, the efficiency is

$$E = \frac{P}{W\,h}.$$

Both P and $W\ h$ may be expressed in horse-power by dividing by 550, but of course the ratio between them remains unaltered provided both are expressed in the same units. In hydraulic problems generally the *rate* of expenditure of energy is directly implied, because the velocity of water enters into the conditions. Thus the product of W and h at once expresses the *power* as a definite number of foot-pounds *expended in a stated time*, and by dividing by the usual constant it may be directly rendered into horse-power. It is not possible to lay too much stress upon the distinction between

energy and the *rate of expenditure of energy or power*, for many writers employ a loose method of dealing with the subject of energy generally, to the confusion of those who are endeavouring to understand the relationship between the two. As we are here dealing with a known mass of water at a stated velocity or head the element of time necessarily comes in, and therefore the results are in power-units, generally foot-pounds *per second*.

The loss of $W h - P$ foot-pounds per second occurs both within the turbine and in the velocity with which the water is discharged therefrom. The latter loss is easily ascertained, being $\frac{1}{2}\frac{W v_1^2}{g}$. By deducting this from the total loss, the other losses, which are not directly ascertainable, may be estimated. The internal losses, due to impact and friction of the water on the vanes, eddies, and vortices in the fluid itself, and friction of the shaft in the journals and thrust-bearing, are the remaining sources of loss which complete the balance between losses and power delivered to the shaft on the one hand, against the total power received, which in some way must be accounted for either as a loss or as useful work on the shaft.

The energy stored in the water where it enters the wheel may be partly in the form of kinetic and partly in potential energy. As a general investigation of the losses of energy does not involve a consideration of the relative amount of each kind, but rather the type of wheel, this will be reserved until the two different types of wheel are considered separately. As to the relative magnitude of the various sources of loss in a turbine, take the case of a wheel passing 3,200 cu. ft. per minute and subjected to a head of 12·2 ft.

(under which head the velocity of freely falling water is 28 ft. per second), and that after passing through the wheel the water is discharged at 7·1 ft. per second, the brake-horse-power as measured on the shaft being 51·5. The loss of power due to the friction of the water within the wheel together with the journal friction of the rotating shaft would be the difference between the total loss and that carried away in the discharge.

	Approximate loss in per cent.
(1) Available power in water is $\frac{3{,}200 \times 62{\cdot}5 \times 28^2}{2 \times 32{\cdot}2 \times 33{,}000}$ = 73·8 h.-p.	
(2) Total power lost = 73·8 − 51·5 = 22·3 h.-p.	30
(3) Power lost in discharge = $\frac{3{,}200 \times 62{\cdot}5 \times 7{\cdot}1^2}{2 \times 32{\cdot}2 \times 33{,}000}$ = 4·7 h.-p.	6·4
(4) Frictional losses of all kinds, 22·3 − 4·7 = 17·6 h.-p.	23·9
(5) Efficiency of wheel = $\frac{51{\cdot}5}{73{\cdot}8}$	69·8

Meissner is quoted by the before-mentioned authority, who allocates the various friction losses comprised under (4) into three classes, as follows :—

	Per cent.
(1) Sum of losses by friction and impact in guide apparatus and wheel	10·5 to 14
(2) Loss by leakage through clearance space between the guide vane ring and wheel	4·5 to 4·5
(3) Friction in air and water of bearings	2 to 3·5
Total losses other than residual	17 to 22
(4) Loss due to residual velocity	6 to 6
	23 to 28

Other authorities give different results than these. The residual velocity may be reduced so that the loss under this head is less than 6 per cent., but it is usually close to this figure. The above figures are for a reaction wheel which, working entirely submerged, renders the friction loss greater than for a turbine which encounters only air resistance as it revolves.

CHAPTER V

VARIOUS TYPES OF TURBINE.

Reaction Mixed Flow Turbine as Used for Low Falls.

The illustration shows a Francis turbine arranged for low falls. This wheel differs but slightly from that employed in the low fall plant which is described in Chapter VII. The guide vanes are hinged and may be turned through an angle sufficient to completely close the apertures between them against the entrance of water to the wheel, or they may be placed so as to afford a full opening for the admission of water. In Fig. 28 they are shown in full lines arranged for the admission of water to the wheel, but when turned into the position shown by dotted lines they act as an ordinary gate to completely shut off the water. As the angle of entrance to the vanes is consequently subject to alteration, the efficiency of the wheel suffers owing to the loss of energy by shock at entrance when the water is admitted at an unfavourable angle. The movement of the guide vanes is effected through the vertical rods A and B (Fig. 27), which are rotated by the sector bolted to the I joists which support the bearing C. This sector is moved by a worm, which in turn receives motion from the governing arrangement. The regulation is necessarily performed against great resistance. The movable vanes possess considerable inertia, and if the water carries impurities, the pivots

are liable to become choked by foreign matter, which adds to the friction and renders sensitive governing a

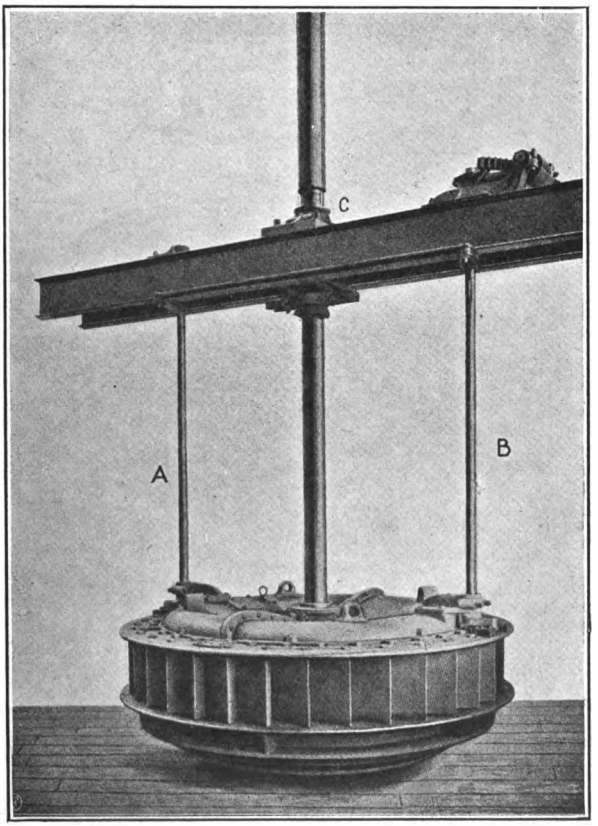


FIG. 27.—Mixed flow turbine as used for low falls.

matter of difficulty. The submerged mechanism cannot be artificially lubricated, and cleaning can only be done by draining the pit, which is not always possible, where the

turbine is in constant use. The weight of the turbine is carried by a thrust bearing at the top of the shaft above the gear by which the power is usually taken off to the horizontal shaft. This arrangement of turbine is especially suitable for falls up to 10 feet. The cast iron ring at the bottom is embedded in the concrete of the arch which divides the head from the tail race, or it may be supported upon a grillage of steel embedded in concrete. In some arrangements a wooden floor is substituted, the spaces between the boards being caulked with oakum, but the weight of the guide ring should nevertheless be carried by steel beams, and to preserve them from corrosion they should be protected from the water by concrete well packed about them.

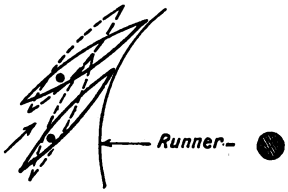


FIG. 28.—Movable guide vanes.

REACTION MIXED FLOW TURBINE AS USED AT NIAGARA.

The arrangement of a 5,500 h.-p. turbine, as installed at Niagara, is shown in the two next illustrations. As this famous installation is one in which the principles of turbine construction have been combined to form a highly successful and efficient plant, it may be used as an illustrative example of the way in which the engineer surmounts the difficulties which oppose his skill in designing water power plants in general, and the type of turbine which was adopted after careful consideration represents one of the best forms of wheel of the class.

The illustration (Fig. 29) is a vertical section through the shaft of the turbine, including the draft tube which is forked

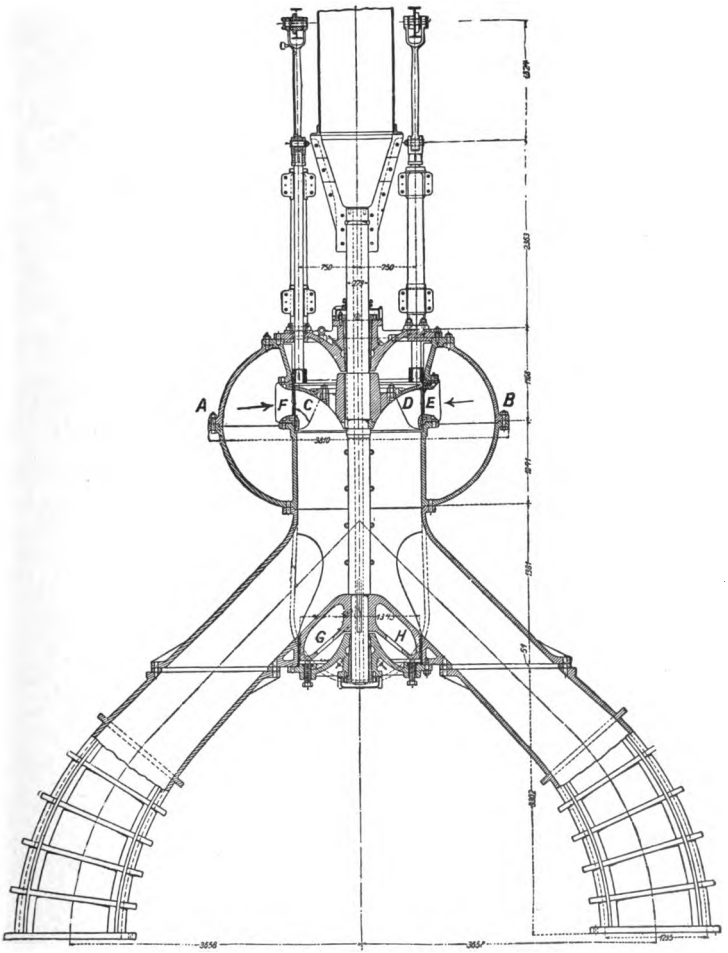


FIG. 29.—Niagara mixed flow turbine, 5,500 h.-p.

beneath the wheel chamber for the purpose of allowing an uninterrupted flow through the tail race tunnel, as otherwise

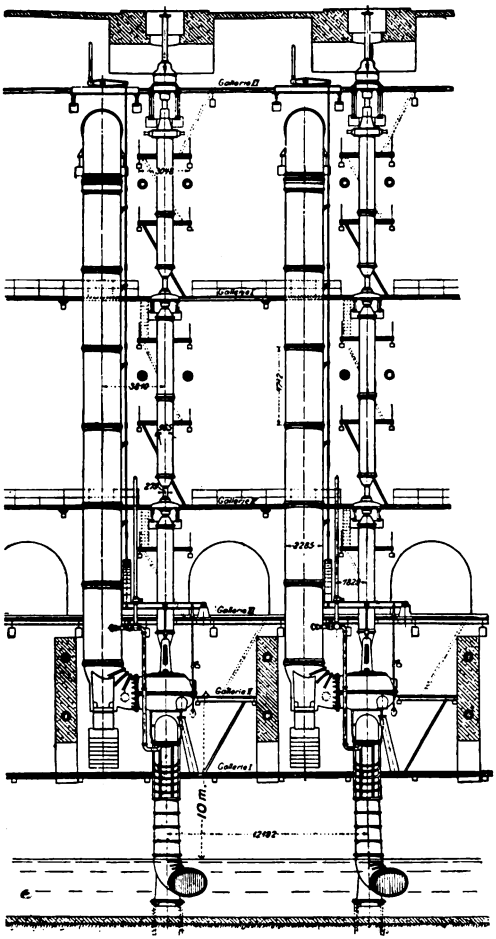


FIG. 30.—Niagara turbines.

the tube passing through the water would offer obstruction. The two branches of the tube are brought down on each side of the tail race and discharge their contents beneath

the surface of the water in the race. The penstocks which convey the water to the wheel are shown in the second illustration to the left of the shafts, and by means of a quarter-turn bend the water is admitted within the casing A B which encloses the wheel C D (the letters are placed on the vanes) (Fig. 29). The guide vanes E F, which completely surround the wheel, admit the water at a fixed angle to the wheel. The runner, which is cast entirely of manganese bronze, is cone-shaped, so that the water is deflected downwards into the draft tube. The principal dimensions of this turbine and setting are as follows:—

Diameter of wheel (runner), 5 ft. 4 in.

Outside diameter of turbine casing, 3,810 mm. (12 ft. 6 in.).

Uniform diameter of penstock from point of exit at canal to turbine casing, 2,285 mm. (7 ft. 6 in.)

Approximate height of centre of turbine above water in tail race, 6 m. (19 ft. 8 in.).

The normal speed of the wheel is 250 revolutions per minute, and under the fall of 44·5 metres (146 ft.) it develops 5,500 h.-p.

The turbine, being a reaction wheel, works with all parts of the casing, wheel, draft tube, and penstock full of water. The draft tubes dip below the surface of the water in the tail race, and thereby the suction becomes effective. The wheel is controlled by a circular bronze gate which is interposed between the ring of guide vanes and the runner. This gate is moved up and down in the direction of the shaft by the governor located in the dynamo room 130 ft. above. Though the gate may be in such a position as to partially screen the wheel from the water, the latter is nevertheless still completely full whatever the position of

the gate may be as determined by the governor. The spaces behind the gate become filled with stagnant water, but no appreciable amount of air is present at any place within the casing. The local conditions required that the dynamos should be located some 130 ft. above the turbines, which necessitated a very long shaft. This shaft is made of steel tubes 965 mm. (38 in.) diameter except at intervals where bearings are located. It is reduced at these places by cone-shaped castings to a smaller steel shaft. The thrust bearing is placed on the top deck, but to relieve this of as much pressure as possible, a large part of the weight of the turbine, shaft, and rotating field of the electric generator is balanced by the upward pressure of the water against a disc G H, shown below the turbine in Fig. 29. Water from the penstock is admitted by a pipe to the space below the disc, and the upward reaction almost balances the weights. A series of stuffing rings on the periphery of the disc prevents undue leakage, and air is prevented from entering the suction tube by the cover plate of larger diameter than the disc. The upward pressure exerted by the water is about 60 tons, but there is a slight difference between this and the weight of the revolving parts so that there may be always a small resultant pressure against the thrust block. The weight of the circular gate and mechanism is balanced so that the governor has only to overcome the inertia of the mass in the process of regulation. The peripheral speed of the outside of the runner is about 70 ft. per second, that of the water under a free fall of 146 ft. being about 97 ft. per second, so that the turbine speed is 72 per cent. of the velocity of free fall. Since this turbine was installed a wheel of 10,000 h.-p.

has been designed, which is the largest unit in the world at the present time.

Girard Turbines on Horizontal Shaft with Partial Admission.

The accompanying illustration (Fig. 31) of two 72-in. horizontal Girard turbines is reproduced by the courtesy of the manufacturers, Messrs. W. Gunther & Sons, Oldham. One of the turbines is shown with the casing removed to disclose the runner vanes. The water enters through the flanged openings and is directed against the wheel from a series of vanes which are below the floor line, and which extend over a small arc of the circumference so that the turbine works as a partial admission wheel. The discharge into the tail race is at the lowest level of the wheel. These wheels drive a textile mill through ropes, the pulley for which is upon the shaft between them. Each of them yields 500 h.-p. under a fall of 520 ft. (158·5 metres) and is 72 in. (1·83 metres) in diameter. These Girard turbines are most efficient when working under comparatively high falls, and may be compared with the Pelton wheel as being used under conditions favourable to both types. They are governed by an independent governor, not shown in the illustration.

The Pelton wheel, which was first used in America, is a form of Girard or impulse turbine in which the vanes or buckets are arranged in such a manner that a jet of water from a nozzle is directed in the plane of the wheel tangentially to the circle described by the mean radius of the buckets. The buckets are constructed of cast iron,

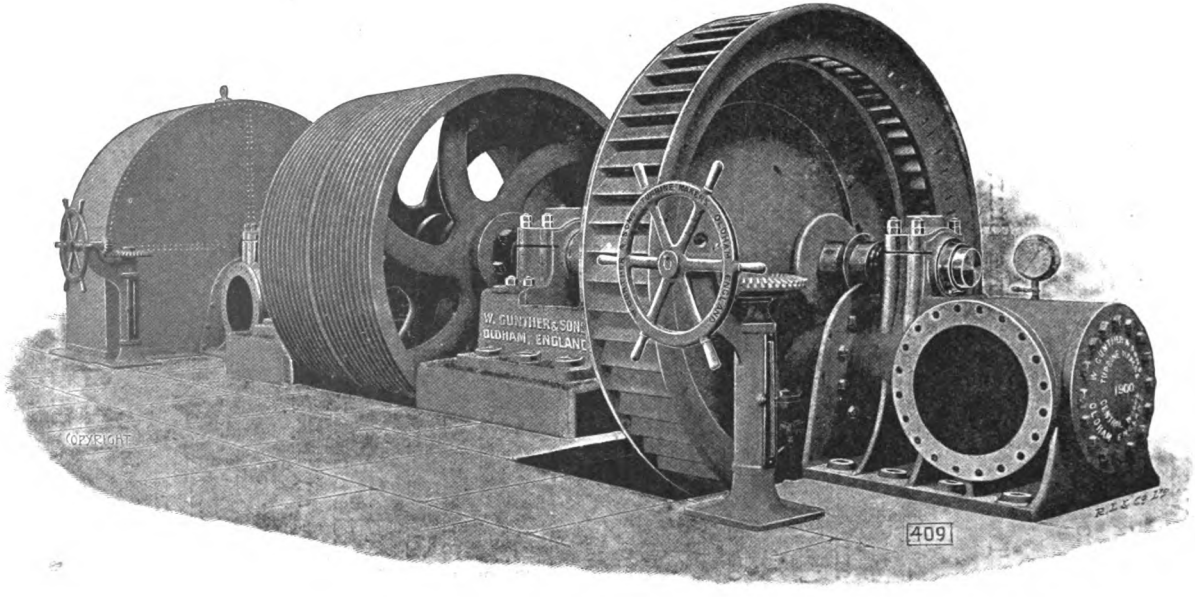


FIG. 31.—Two 72-inch Girard turbines on horizontal shaft.

cast or pressed steel, and are made double, so that the jet of water, impinging on the central edge, is deflected in both directions. The angle through which the water would be deflected, were the wheel held stationary, would be almost 180°, as the outer lips of the buckets are turned back so

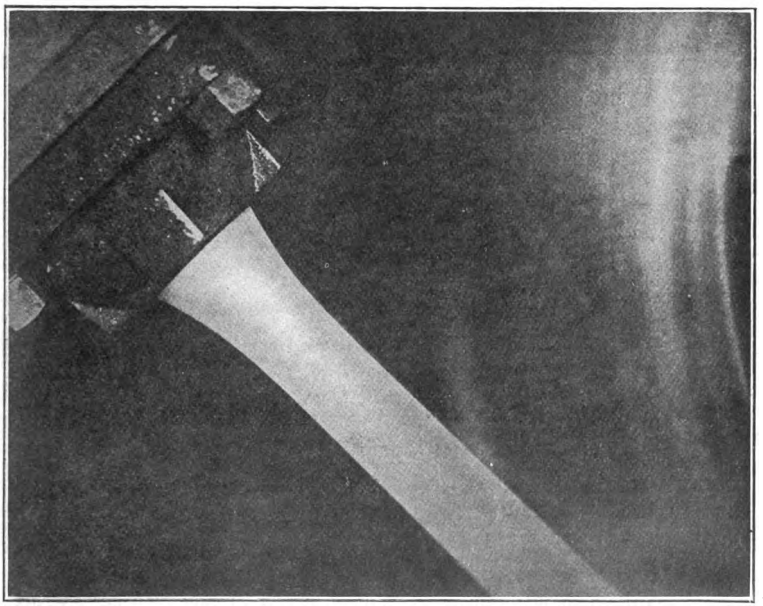


FIG. 32.—Jet issuing from nozzle at high velocity.

that a tangent to them has only a slight inclination with the direction of the jet. With the wheel in motion the direction that the water takes on leaving the bucket is more nearly axial; but if the lips of the buckets were turned completely back the water would be discharged in the plane of the wheel, and the direction would not be affected by the speed.

The minimum *absolute* residual velocity, corresponding to the highest efficiency, is zero when the speed of the wheel is one-half that of the absolute velocity of the jet. Such a condition is impossible in practice, as the water must be endowed with a residual velocity to clear the buckets. Especially for mining districts and in outlying places where water is plentiful and the cost of transportation and erection of a turbine renders it inexpedient to install a more elaborate plant, the Pelton wheel is a very useful form of motor. It is constructed in small sizes for rough work on a self-contained frame, and the only works necessary for the installation are the water pipe and a trough for the discharge. The range of head for which these wheels are suitable is from 50 ft. upwards, though at this minimum limit other forms of wheel become serious competitors. Even at 100 ft. it is very doubtful if this wheel can be considered to be good engineering, unless efficiency is of little importance, for the claims of very efficient reaction turbines have to be argued away to justify their use under such conditions.

The most powerful wheels of this type that have been built are those of the Rio das Lazes hydro-electric station of the Rio de Janeiro Tramway Light and Power Company. These wheels are 9,000 h.-p. each, and they operate under a head which ranges from 950 to 1,000 ft. Each wheel is supplied by four needle nozzles, and they run at 300 revolutions per minute. The efficiency is estimated at 82 per cent. Each wheel receives water through a 36-inch welded steel pipe which varies from 0·4 to 0·7 in. in thickness, depending upon the head. (See Appendix C.)

The shape of the nozzle from which the water is

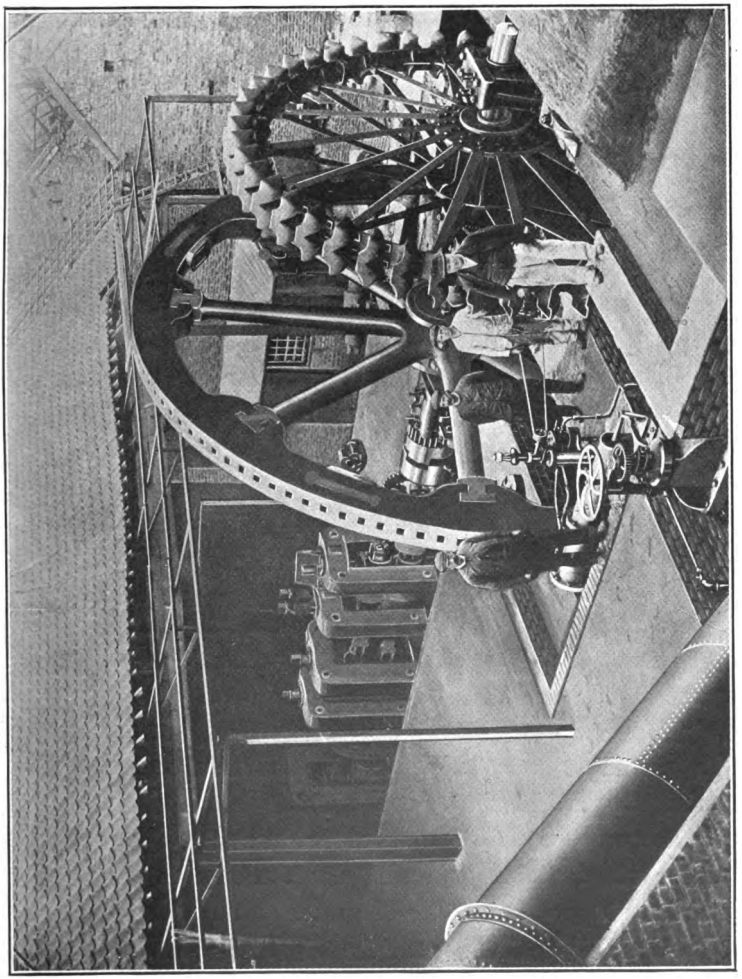


FIG. 33.—Pelton wheel 21 ft. in diameter driving tin plate rolls.

discharged is important, as it is essential that there shall not be any dispersion of the jet. The water should issue in a solid jet as shown in Fig. 32, which is a photograph kindly

placed at my disposal by Mr. H. E. Warren. It will be seen that the jet suffers considerable contraction in area close to the mouth of the nozzle.

The wheels manufactured by Messrs. Gilbert Gilkes & Co., Ltd., Kendal, range in size from 6 in. up to 21 ft. in diameter, and Fig. 33, for which I am indebted to the firm, shows the construction of one of the largest. This 21 ft. wheel, designed to develop 200 h.-p. with a 98 ft. fall at 36 revolutions per minute, drives tin plate rolls by direct connection. The large diameter results in slow speed so that gearing is avoided between the motor and machine. The smaller wheels are not as efficient as the larger, nor is the same wheel as efficient under low as under high heads. Two wheels, one 8 in. diameter and the other 72 in., will have a difference in efficiency of almost 2 per cent. under the same low head, while a wheel of the latter size will have a distinctly better efficiency under 700 ft. head than, say, 200. For extremely high heads, such as that of the Lake Tanay installation or the Manitou plant (3,018 ft. and 2,417 ft. respectively), the efficiency is somewhat less than that attained with moderate falls owing to the enormously increased friction losses. There is consequently a medium head, which experience shows is about 350 metres (1,148 ft.), at which a wheel of this type will give a maximum efficiency of about 80 per cent. The diameter of the wheel should be as large as possible to attain the best results, for the buckets are placed more advantageously in the stream, as they move with less angularity to the direction of flow. For heads of 200 to 1,500 ft., with a freedom of choice as regards the diameter of the wheel, it should be possible to

obtain an efficiency at full load of 75 to 77 per cent. with these wheels when constructed by firms of repute. It is very doubtful if efficiencies exceeding 77 per cent. are often attained by them, though extravagant claims are made by some American makers. The difficulties of testing wheels render statements concerning efficiency safe from confirmation.

Testing Turbines.

The recorded tests upon hydraulic turbines are very few considering the thousands of wheels which are in continual use throughout the world. Even though efficiencies and output are specified by the purchaser's engineer when ordering a turbine, but few owners care to subject themselves to the expense of a test made by experts under scientific conditions, to prove the worth of the turbine when installed. If the purchase was a steam engine, a pair of indicators, gauges, and perhaps a Prony brake, would suffice to obtain accurately the characteristics of the motor, but the apparatus required for testing a turbine is more elaborate, and the expense entailed is much greater.

The late Mr. J. B. Francis for many years conducted elaborate tests upon turbines which still remain the most valuable information on the subject, and the records of the Holyoke testing station are especially interesting in view of the variety of wheels which were subjected to critical examination. As far as the writer is aware, this permanent testing station upon the Connecticut River at Holyoke, Massachusetts, is the only thing of the kind in the world, if we except the hydraulic plants which usually form part of well-equipped mechanical laboratories in our educational isntitutions, but which are necessarily limited for the purposes

of testing. The Holyoke testing flume is arranged so that a wheel may be fitted into it, supplied with water under a prescribed head within the limits of fall of the river, and the power measured by a brake. The amount of water discharged is measured accurately, and, from first to last, valuable information regarding the wheel is obtained. The flume was originally designed for testing wheels for a local company which controlled the power rights upon the river, but soon it became a recognised testing station throughout the country, and the manufacturers of turbines resorted to it to prove their wheels.

The tests applied to a turbine are similar to those to which other prime movers are subjected, the principal one being efficiency, or ratio of power developed to power supplied. This, in a turbine as generally understood, is the ratio of the brake horse-power, as measured on the driving shaft, to the theoretical power of the water falling through a height equal to the head. As the power in the falling water depends upon the flow, it is essential that an exact quantitative measurement be made of the water passing through the wheel. The necessity for such a measurement forms one of the chief obstacles to testing, as it involves weirs and special appliances which are not often readily available. The power developed upon the shaft of the turbine may be measured by a mechanical brake, of which there are several kinds, or, if there be an electrical generator with known efficiency, it may be directly measured electrically. As nearly all electrical engineers require shop tests of generators to be made by the manufacturers, the efficiency is generally known with sufficient exactness from the curves supplied from the testing room. These curves show the efficiency

for different percentages of load, and from them the power required to drive the generator for a given output may be readily obtained. Thus if W be the output in kilowatts of a generator coupled to a turbine, as measured directly by an ammeter and voltmeter, and e be the efficiency of the generator, the power required in horse-power on the shaft of the turbine will be $p = \frac{W}{0{\cdot}746 \times e}$, and if P be the theoretical-horse power in the water as calculated from measurements on the flow, and E the efficiency of the turbine, we have:—

$$E = \frac{p}{P} = \frac{W}{0{\cdot}746 \times e \times P.}$$

As an example, a 300 kw. electric generator, directly coupled to a turbine, has an efficiency at full load of 89·5 per cent. The horse-power necessary to drive it so as to obtain an output of 300 kw. is therefore

$$p = \frac{300}{0{\cdot}746 \times 0{\cdot}895} = 449.$$

The turbine passes 180 cu. ft. of water per second, works under a 28 ft. fall, and the theoretical horse-power is therefore

$$P = \frac{180 \times 62{\cdot}5 \times 28}{550} = 573.$$

The efficiency of the turbine is therefore

$$\frac{p}{P} = \frac{449}{573} = 78{\cdot}4 \text{ per cent.}$$

The efficiency of the entire unit is the product of the efficiencies of both turbine and generator or $78{\cdot}4 \times 89{\cdot}5 =$

70·2 per cent. An efficiency of 76 per cent. can be obtained from low fall turbines at full load, *i.e.* at best speed. Referring to the installation described on p. 135, the full load efficiency of the wheel is 76 per cent., of the electric generator 90 per cent., and the loss in transmission from the generating station to the place of distribution of the current 7·2 per cent. Allowing 10 per cent. loss in the gearing and shafting between turbine and dynamo, the efficiency of the installation

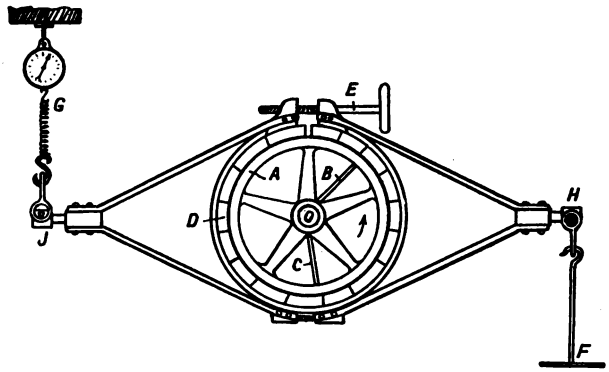


FIG. 34.—Prony brake.

would be :—0·76 × 0·90 × 0·90 × 0·92·8 = 57 per cent., *i.e.* more than half of the actual energy in the falling water is available at the distributing board one quarter of a mile distant, when the turbine is working at full load.

The simplicity of the electrical method of obtaining the efficiency of a turbine cannot always be hoped for, and if the wheel is to be used for other machinery, the output must be measured by some form of absorption dynamometer, the simplest type of which is the Prony brake illustrated in Fig. 34.

It consists of a cast iron wheel A with a hollow cored rim through which a constant stream of cooling water is made to flow, B and C being the pipes connecting the rim to the hollow shaft, through the end of which the water is admitted. As all the power is converted into heat at the periphery of the wheel, an ample supply of cooling water is essential by which the heat may be carried off. The periphery of the cast iron wheel, which is turned true, is surrounded by a series of hard wood blocks D, and these are pressed against the wheel by the wrought iron clamp bands which may be tightened by the screw and hand wheel E. If the wheel is caused to revolve in the direction of the arrow it will tend to lift weights placed in the scale pan F and to stretch the spring G, the tension of which may be measured on the spring balance dial. To maintain the arms O H and O J in their original position against the turning effort, it would therefore be necessary to add weights to the scale pan F until the tendency to turn, as produced by the friction between the rim and wooden blocks, is balanced. When this condition is attained, the weights and tension in the spring together with the speed of the wheel give a measure of the power, which may be readily deduced. For if W be the loads in pounds necessary to hold the balance against the turning moment, the power developed for N revolutions per minute is :—

$$H.\text{-}P. = \frac{W \times 2\pi r N}{33{,}000}$$

The measurement of the power does not involve the size of the brake wheel A, but for brakes to absorb 200 h.-p. up to speeds of 300 revolutions per minute it would be about 3 ft. 6 in. to 4 ft. in diameter, and the distance r

(O J = O H), or effective radius at which the loads are placed, would be about 4 ft. 6 in.

Example.—A turbine running at 198 revolutions per minute delivers power to an absorption dynamometer in which r = 4 ft. 6 in., and the loads are 487 lbs. Therefore

$$H.\text{-}P. = \frac{487 \times 2\pi \times 4{\cdot}5 \times 198}{33{,}000} = 82{\cdot}6.$$

These dynamometers are unsatisfactory under rapidly fluctuating loads, for the beam becomes unsteady and oscillates. Moreover the wheel must run very true, or else a correct reading cannot be got. Plates of sheet iron screwed to the sides of the wooden blocks and overlapping the rim of the wheel are usually fitted to prevent motion parallel with the shaft. This dynamometer is similar to the electric generator in that the power is absorbed, but there are other forms in which the power is measured and transmitted, and which may therefore be temporarily applied to a turbine at work, but, compared to the absorption type, they are seldom employed.

A new form of power-measuring device, by which the power passing through the shaft of a marine turbine may be measured, depends upon the twist which a shaft experiences when power is transmitted through it. By means of a delicate electrical appliance this twist in a long shaft may be measured, and the power calculated. It is, however, necessary that a considerable length of shaft be taken to show the minute torsional deflection, and as long shafts are not often required in hydraulic turbine installations it is doubtful if it has any application to this department.[1]

[1] For an account of the Denny and Johnson Torsion Meter, *see Engineering*, April 7th, 1905.

That the high efficiencies claimed by turbine manufacturers are attained by some of the leading firms, the

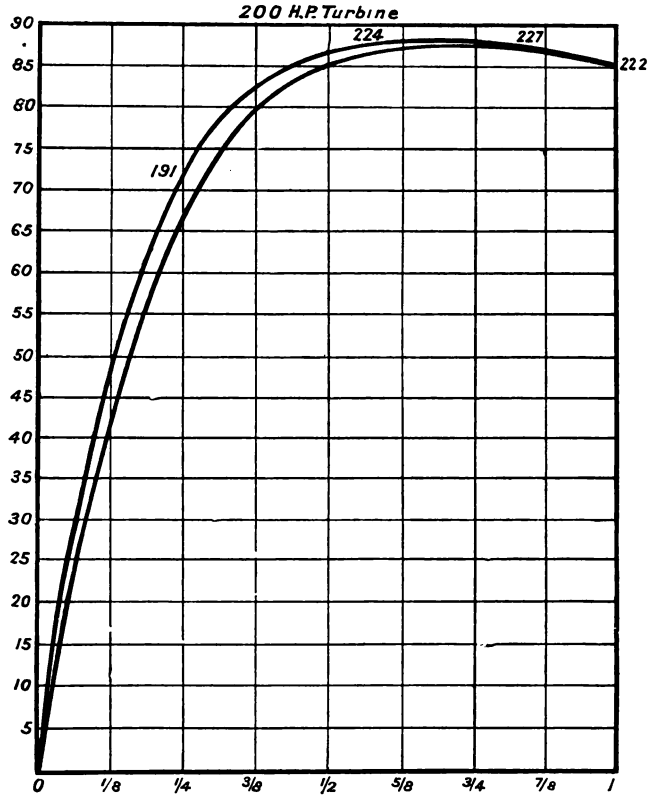


FIG. 35.—Tests on 200 h.-p. turbine.

records of actual tests show. The Swiss firm of Messrs. Theodor Bell & Co., Kriens-Luzern, have supplied the writer with some records, from which the curves shown in Fig. 35 are reproduced.

The two curves illustrate the variation in the efficiency of the wheel when working (1) at the most favourable speed; (2) at the constant normal speed. The figures on the curve are the revolutions per minute. It will be seen that

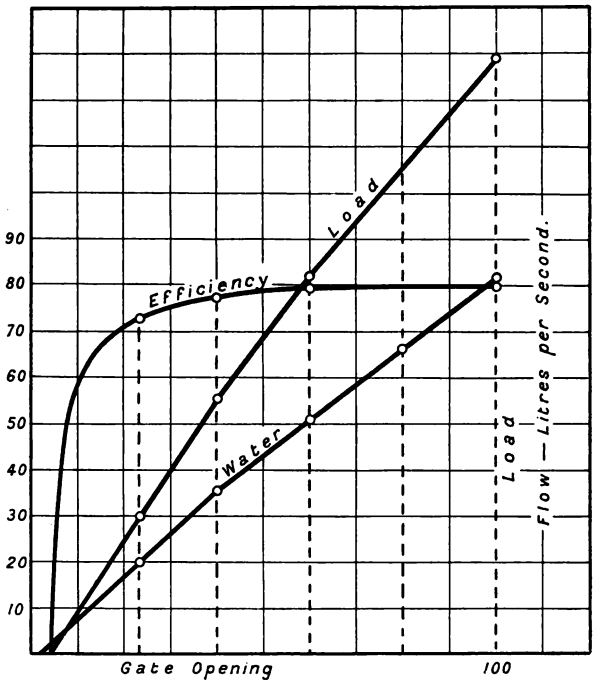


FIG. 36.—Tests on Pelton wheel.

an efficiency of 88·5 per cent. is attained at the most favourable speed between a gate opening of 0·625 and 0·75, while for the same gate opening at the constant normal speed it is almost 1 per cent. less, and for small gate openings the difference is more marked. The maximum efficiency is, in

this case, not coincident with full gate, but the normal working of the turbine is between 0·625 and 0·75 full gate, and slight departures therefrom effect the efficiency very little. Electrical driving requires a constant speed, so that it is impossible to allow a wheel to revolve at the best speed for all positions of the gate. The lower curve would therefore be the efficiency curve in such cases.

The efficiency, load, and water supply to a Pelton wheel

Per cent. of full gate opening.	23 per cent.	40 per cent.	60 per cent.	70 per cent.	100 per cent.
(1) Fall in metres . .	307	305	304	301	298
(2) Flow in litres per second	196·5	353	503	653	813
(3) Theoretical horse-power	804	1,436	2,039	2,621	3,230
(4) Electrical output, kw. .	369	747	1,134	1,453	1,815
(5) Efficiency of generator, per cent.	84	91·4	94	95·4	96
(6) Horse-power of turbine	597	1,111	1,640	2,071	2,571
(7) Efficiency of turbine, per cent.	74·25	77·4	80·4	79	79·6

constructed by Messrs. Theodor Bell & Co. are shown in Fig. 36, and the table gives the same particulars, from which the curve is plotted. It will be seen that the wheel maintains an almost constant high efficiency from a gate opening of 60 per cent. to full gate, and the horse-power of the turbine increases almost in direct proportion to the flow, as may be seen from the straight line showing the load. The power developed is proportional to the discharge as shown by the straight line marked "water," which gives

the discharge in litres per second. The maximum efficiency of the wheel is attained with a gate opening of 60 per cent., and the combined efficiencies of turbine and generator, 80·4 × 94 = 75·6, represents the over-all efficiency of the installation from theoretical hydraulic power to electrical energy put into the line by the dynamo. Line (3) in the table shows the hydraulic power, which is directly derived from (1) and (2) by dividing their product by 75 (75 kg.-m.

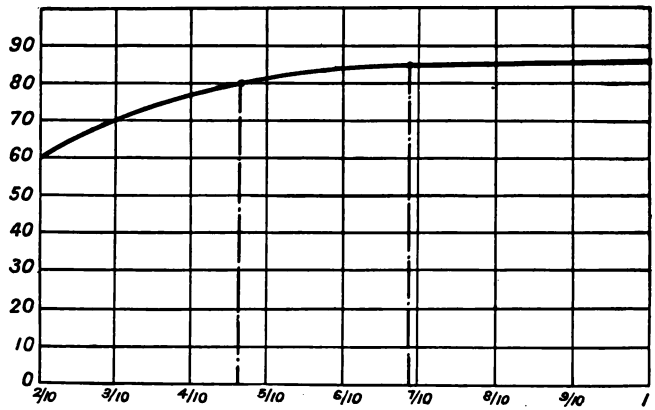


FIG. 37.—Tests on double Francis turbine.

per second in a *force de cheval*). The output of the generator in kilowatts is given in (4), and the horse-power of the turbine is obtained by dividing the equivalent horse-power in line (4) by the efficiency of the generator. The *force de cheval* is equal to 735·5 watts, thus, taking the electrical output at 60 per cent. gate opening, which is 1,134 kw., this is equivalent to 1,134/0·7355 = 1,543 h.-p.

A series of tests were made by Professor F. Prasil upon a turbine constructed by Messrs. Escher, Wyss & Co., which

show what may be obtained from a double Francis turbine with horizontal shaft supplied with water under a head of about 6·7 metres (22 ft.) and yielding a power of 200 h.-p. The power was measured by an absorption dynamometer in the form of a Prony brake, one metre in diameter and 25 cm. wide, and the flow was obtained from velocity meter records. The meters were immersed in the water at various points across the section of the channel and the mean flow was calculated from the readings thus obtained. The constants for the meters formed the subject of previous inquiry, and were determined with apparently satisfactory accuracy. From the various tests it was found that the efficiency of the wheel, when approximately 200 h.-p. was being absorbed by the brake, was 85·23 per cent. The mean speed was 141 revolutions per minute and the mean head 6·67 metres (21·9 ft.). The curve (Fig. 37) shows the variation in the efficiency with the gate opening.

CHAPTER VI.

CONSTRUCTION OF WATER-POWER PLANTS.

To cover all the varieties of construction works for water-power plants that have to be adopted to suit local conditions would be impossible within the limits of the few pages at our disposal; consequently a few of the more general and common features of the construction of works common to the majority of plants can alone be dealt with. As nearly all the small water-powers, especially in England, are obtained by damming up a river so as to produce a fall, the construction of dams will perhaps repay a little attention, as an improperly constructed dam may be a constant source of trouble, not to say of danger.

The design of high dams involves calculations based upon assumptions as to the nature and incidence of the stresses set up by the water pressure. These assumptions, as recent discussions of the subject show, are not universally accepted, and may be at variance with what actually occurs in a mass of material acted upon by conjugate forces. Some engineers, whose authority is unquestioned, contend that the accepted principles for the construction of dams are faulty, and that the factor of safety ascribed to the section by current principles of design does not exist in the structure. Into these and kindred hypotheses, upon which no

direct proof is forthcoming one way or the other, we cannot enter; suffice it to say that many high dams all over the world have not given way, though they are designed upon principles which some would, in the light of experimental evidence, condemn as faulty, if not as actually dangerous.

As a rule dams for impounding small heads of water are constructed very much wider and heavier than any accepted theory of stability would pronounce to be sufficient. Leakage, liability to destruction by floods, and possibly the necessity for a right of way along the top, determine in many cases the width rather than the resistance to overturning by hydrostatic pressure. The question of leakage is especially important, and the cross-section may be considerably modified from that which theory would assign, lest there be insufficient width at the base to prevent the seepage of water down-stream, for leakage, once begun, quickly grows, and involves the speedy disintegration of the material, especially if the facing of the dam be backed by earth or rubble.

It can be easily shown that, if a vertical wall sustains a pool of water, thus acting as a dam, the total pressure of the water on a vertical strip of unit width, irrespective of the extent of the pool up-stream, is half the depth of the water in feet multiplied by the weight in pounds of a cubic foot of water multiplied by the area of the strip. For example, a wall 5 ft. high sustains water, the level of the water being at the top of the wall. The total pressure on a strip of the wall 1 ft. wide is therefore $\frac{1}{2} \times 5 \times 62{\cdot}5 \times 5 = 781{\cdot}25$ lbs. Expressed as a formula, in which l is the length of the strip along the wall, h the head of

water, and γ the weight of the cubic unit of water, the total pressure—

$$P = \frac{1}{2} h \times \gamma \times l h = \frac{\gamma}{2} l h^2.$$

The total pressure is, therefore, proportional to the *square* of the head of water, so that the tendency for a wall of uniform section to be overturned by the water pressure increases rapidly with the height of the wall. If h and l are expressed in metres, and P is measured in kilogrammes, since one cubic metre of water weighs 1,000 kgs.

$$P = 500\ l\ h^2.$$

The total pressure upon the wall can be resolved into a single force which acts at two-thirds the total depth of the water, measured from the water level downwards. If the dam were composed of a solid mass capable of being overturned about the down-stream edge or "toe," this force would be considered alone. As, however, dams are made up of materials cemented together and capable of being broken along vertical and horizontal planes, a correctly designed dam must be proportioned so that it will resist splitting along any horizontal plane, and as the pressure of the water increases from the top downwards the cross-sections therefore must also increase, and this gives rise to a curved outline on the down-stream edge. This curved outline can be arrived at by calculation to give a desired factor of safety; but dams are never constructed to the exact mathematical curve for structural reasons; and for low dams, such as the development of small water-powers require, purely arbitrary sections are chosen. It is not necessary to follow the theory of the design of structures of this kind here, which only apply to high dams where a

saving in the weight of masonry is specially important. For low dams structural conditions render a departure from any theoretical plan necessary, and it is with such that we especially deal at present. It may, however, be noticed that dams are of two kinds—those which offer resistance to the water by their dead weight alone and adhesion to the foundations, and those which are arched up-stream so as to develop resistance by a thrust at the abutments on each bank. This latter principle is only utilised for high dams, and mainly on rivers with precipitous banks to which the arch thrust can be transferred. For low dams the gravity principle is alone used, and it is this kind of structure that we are now considering, in which the resistance required against displacement is provided by the weight and adhesion of masonry or earth to the foundations, which forms a homogeneous mass when carefully put together.

As so much depends upon the nature of the material used, whether earth or stone, which varies in one place from another, and of the class of cement and method of gauging, it is not possible to lay down any rules which are applicable in all cases.

Dams may be classified as follows:—

(1) Earth dams.

(2) Earth with masonry facing.

(3) Masonry, which may either be ashlar, or rubble with a facing of cut stone.

The earth dam is perhaps the oldest of all forms of dam, and is, by reason of its comparative cheapness and facility of construction, becoming a common form of structure for reservoirs designed to impound water at low heads. At

the same time it has been employed for comparatively high heads, as for instance at the Yarrow dam of the Liverpool waterworks, which is some 90 ft. high. In the case of this dam the foundations had to be carried 97 ft. below the original surface to obtain a layer upon which the superstructure could rest which would be entirely impermeable to water. Such dams are constructed of clay "puddle," which has to be tamped down hard, so that the mass may be impervious to water. Dry clay will absorb considerable quantities of water, which varies according to the nature of the soil, and which is estimated by different authorities at 33 to 60 per cent. of its weight.

Mr. Burr Bassell[1] states that "there are in use to-day 22 earth dams exceeding 90 ft. in height, and twice that number over 70 ft. in height. Five of the former are in California and several of these have been in use over 25 years." Furthermore, he avers that "he fails to appreciate the reason for limiting the safe height of earth dams to 60 or 70 ft." The earthquake in California (April, 1906) brought out the superiority of the earth dam to withstand shocks of great severity, for though the great fault line, which formed a plane of shear in a N.W.-S.E. direction, passed directly across a high earth dam, the structure was not severely injured, as it would have been if it had been constructed of masonry, for in that case the fracture would have rendered it ineffectual for its purpose.

The form of an earth dam, as usually constructed, is that of a truncated triangular section with a slope of about two to one on both faces, but as the requisite degree of slope varies with the quality of the material

[1] "Earth Dams" by B. Bassell.

(being naturally steeper for a stiff clay), the cross-sections of such dams display a great variety of shapes depending upon the caprice of the designer. The surfaces of earth or puddle dams are best covered with a pitching of stones and may be covered with grass, as in the retaining banks for reservoirs, though in this case they form only part of a composite embankment to retain the water. The nature of the material is an important condition to the success of earthen dams, for some clays will not pack sufficiently hard to prevent the seepage of water through them. Engineers are not agreed as to the best qualities to be looked for in material for an earthen dam. It is suggested by some that a certain proportion of sand in the earth is beneficial, and earthen dams have been constructed with mixtures of earth and sand containing 20 per cent. of the latter ingredient.

Masonry Dams.

Notwithstanding the extra cost of stone dams over those constructed of clay, their increased durability and comparative immunity from destruction by floods or freshets, besides the valuable feature of providing a spillway where required, render them the best form of structure for holding back a few feet of water for a small plant. Neglecting for the moment the adhesion or anchoring of a masonry wall to a foundation, and supposing that the dead weight of masonry alone were to be employed to resist the hydrostatic pressure, it is easily shown that a rectangular wall with a height of less than 2·8 times the width will withstand overturning. To prove this, take 165 lbs. per cubic foot for the approximate weight of granite or limestone masonry,

and if d is the width of the wall (Fig. 38) and h the head of water sustained by it, the overturning moment is, for each unit of length of wall—

$$M = \frac{62{\cdot}5}{2} \times h^2 \times \frac{h}{3} = 10{\cdot}4\ h^3$$

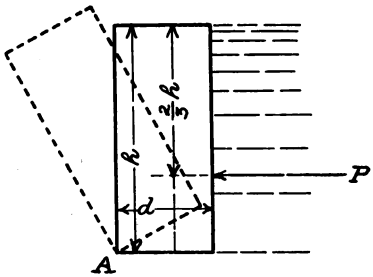


FIG. 38.—Overturning by hydrostatic pressure.

The moment of resistance offered by the dead weight of the wall to overturning about the downstream edge A is, for a unit length—

$$M_1 = 165 \times d \times h \times \frac{d}{2}$$

$$= 82{\cdot}5\ d^2\ h.$$

Equating these two moments we have—

$$10{\cdot}4\ h^3 = 82{\cdot}5 \times h \times d^2$$

$$h = \sqrt{7{\cdot}93}\ d \quad \text{or} \quad h = 2{\cdot}8\ d.$$

This shows that a rectangular wall of masonry in which the height is less than 2·8 times the width will resist overturning by water pressure, whatever the actual dimensions may be. Such a wall, being unnecessarily thick at any section above the base for withstanding the hydrostatic pressure at that section, may be reduced in section, and thus a considerable weight of material saved. Other considerations than overturning enter into the problem, especially for high dams where the pressure produced upon the toe of the dam by the resisting moment, together with the dead weight of the masonry, may exceed the crushing

strength of the material, and it is cases of this kind that give rise to problems of special interest in the design of large dams. The tendency of the dam to shear along a horizontal section, or to slide bodily down stream under the action of the water pressure, is resisted by the friction and adhesion of the masonry to the foundation, but this is ordinarily so great that it is unusual for dams to fail in this manner, especially as the width at the bottom is made greater than would for this reason be necessary, for the purpose of preventing leakage. The illustration on page 134 shows a dam which can be constructed cheaply, and which, at the same time, serves as a spillway over the top. The face of the dam is constructed of pointed ashlar masonry, and the back is filled in with rubble, which in this case was well packed down so that the water passing over it would not cause displacement of the stones. The foundations of the wall are carried down about a foot below the bed of the river, and the ratio of width of the ashlar wall to the height is something less than two. Dams of this kind require occasional re-pointing at the face to keep them watertight. The experience of the writer points to this as being the most satisfactory dam for low falls. Other kinds of structures may be used, some of which are made of wood, but which are of a less permanent kind. Two lines of sheet piling with the intermediate space packed with earth and stones forms a good dam, though again of a less permanent description. The necessity for flood-gates and sluices renders the masonry dam particularly adaptable, as they may then be built into the work.

Concrete is a valuable material for the construction of dams and walls, and as it can be laid under water, the work

of construction is often facilitated by its use. For the construction of such the following mixture is recommended :—

Portland cement . . .	One part.
Clean sharp sand . . .	Two parts.
Crushed stone (2 ins.) . . .	Five parts.

In order to save expense in building heavy walls, which would thus take a large amount of concrete if only this mixture was employed, it is good practice to use large stones with the above mixture well tamped about them. The stones forming the concrete proper ought not to be more than 2 ins. along any dimension. This allows the concrete to be thoroughly mixed and thrown into the trench by hand. The crushed stone contains such a large proportion of small chips that the mixture is sufficiently plastic to fill up the interstices completely and to make the wall watertight. In fact, it has been found that the large proportion of very small chips and particles of granite in the aggregate as it comes from the crusher is sufficiently great to render the reduction of the above proportion of sand possible, which (especially if the sand is dirty) is beneficial, as it is replaced by a better material. It is not easy at places far from the coast to obtain really clean sharp sand, and an adulteration of clayey matter is fatal to the proper setting of the concrete. For the construction of wheel pits and works subject to the action of water concrete is also a very excellent material, and by making use of the compressive strength of concrete combined with the tensile strength of steel, beams and turbine pit floors may be constructed in a substantial manner.

Turbine pits may be constructed of wood, and often are, for small wheels working under a low head. The

planks forming the back and floor of the pit ought to be 6 ins. × 3 ins., tongued and grooved, and the joints packed with tarred oakum, after which the surface is then served with a coating of hot tar. This helps to protect the wood from warping; but it is necessary to use well seasoned timber, so that the joints will not gape after being some time in service. The weight of the wheel should be taken off the floor by approved arrangements as described on page 98, but the casing and guide vane rings have to be carried by the floor, which is supported in the usual manner by steel or wooden joists, all of which ought to be served with tar to protect them from the water. The opening in the floor for the passage of the draft tube into the tail race is a source of weakness in any form of wood construction, so that it is better practice to construct the floor of the pit as an arch or floor of ferro-concrete, while the water back may be made of wood, which is sustained against the hydrostatic pressure by either wooden or steel beams, for which purpose old railway iron is very well adapted.

The best floor for turbine pits is constructed of ferro-concrete, which can thus be made flat, and the head room necessary for an arch is consequently saved. The thickness of the concrete ought not to be less than one-twelfth of the greatest span. For example, a turbine pit of 25 ft. × 20 ft. ought to have a thickness of floor (which includes the reinforcement as the bars are imbedded in the concrete) of not less than 2·1 ft., for though the safe thickness of floor will depend upon the load to be carried, deflection must be guarded against, and a floor which would be calculated to safely sustain the load might deflect to an extent sufficient to cause serious trouble with the setting of the turbine.

In calculating the amount of reinforcement necessary, the load may be assumed to be that due to the weight of the water at the maximum height to which it can ever rise in the wheel pit, and by taking such a load evenly distributed over the area of the floor and adding the weight of turbine casing and guide wheel which rests upon it, and assuming this total load to be evenly distributed, the correct proportion can be given to the materials composing the floor.

One of the chief requisites for concrete employed for such purposes is that it shall be watertight, and to attain this end the sizes of the aggregate comprising it must not exceed a certain size. The laws governing the impermeability of concrete were fully discussed and elucidated in an excellent paper contributed to the American Society of Civil Engineers by Messrs. Fuller and Thompson.[1] They point out that both theory and experiment prove that mixtures which give the greatest density when dry do not necessarily give the greatest density when mixed with cement and water. To illustrate this point they state that a cubic foot of fine or of coarse sand when dry will weigh almost exactly the same, whereas when mixed with cement and water the mortar made with the fine sand will occupy a bulk approximately 20 per cent. greater than that made with the coarse sand. As a general result of their investigations they found that strength and impermeability to water go together, and the greater the proportion of cement the more watertight the aggregate becomes. Another conclusion arrived at was that concrete composed of sand and gravel, in which the grains were rounded, was, for the same

[1] Proceedings "American Society of Civil Engineers." Vol. xxxiii., Nos. 3 and 5.

proportion of cement, less permeable than that made of broken stone and screenings, also the permeability decreases materially with the age of the concrete.

The sluices and head gates most suitable for small installations are constructed of wood. A good form of construction consists of two uprights of 8 in. × 8 in. or 12 in. × 12 in., grooved or notched to allow the gate to slide up and down. The latter is a batten panel, which is operated through a rack and pinion geared to a crank through a worm and wheel, for by the use of such gearing the gate will maintain itself in any position. Gates of this kind may be constructed and worked by one man easily up to widths of 12 ft., but for wider openings than this it is advisable to make them smaller, by putting two gates to the opening separated by a post. The obstruction to the water by a post in the race ought to be avoided if possible, but the gates become too cumbersome for hand working, and with the pressure of water against them, difficulty may be experienced in moving them without the aid of other power. On the large hydro-electric installations power applied through electric motors is employed to operate gates.

CHAPTER VII.

WATER-POWER INSTALLATIONS.

So various are the conditions under which water-powers may be found in those countries to which the engineer is turning for the future supply of this natural power, that it would be impossible to do more than touch upon the main characteristics of installations, for they are designed to meet the necessities of each particular case. The subject may therefore be best dealt with by describing installations where different conditions of flow and head of water prevail and which give rise to special forms of machinery and appliances. The range of conditions under which water-power may now be profitably developed is ever widening, due to the improved apparatus at the disposal of the engineer. Before the introduction of the turbine in anything like the present form, the mill stream, falling through a few feet, represented the usual conditions from which there could not be a great departure, and which left but little choice for the engineer who had to select a suitable wheel for utilising the power running to waste, but now the range of utilised head ranges from more than 3,000 ft. down to 2 ft. A complete discussion of the subject at the present time would include not only such small powers as may be found upon the rivers of England, and gigantic powers of medium fall such as Niagara and Victoria Falls, but also of powers which make up for a

very small discharge of water by an extremely high head such as the Lake Tanay installation in Switzerland or the Pikes Peak plant in Colorado. The smaller powers from which 100 to 500 h.-p. may be obtained are those which in the future will play an important part in the economy of power generation. It is true that Niagara, and probably the undeveloped Victoria Falls and the great Falls of the Iguazu in Brazil, will be the most important instances of the kind in the world, as they are natural powers of surpassing size, but the method of attacking these vast stores of power is essentially the same as that for many sites, where, with the same head the flow is not so great, and, as far as Niagara is developed, it may be considered to be an aggregation of units which find a parallel elsewhere, but which by their size, and the natural attractions of the locality, have gained a large measure of popular interest.

Nature supplies water-power in a variety of ways, some of which require, on the part of the engineer, a vast amount of work to be done before they can be reclaimed to use, while others are almost ready made and only require the installation of the machinery. Of the former kind there are rivers with a fall which is spread over a long stretch and which can only be utilised by the construction of a costly dam which causes the inundation of a tract above it and consequent storage of water in a reservoir. Other natural powers, such as that of Niagara, are formed by the precipitous descent of a river bed in one great step over which the water tumbles. The development of this kind of water-power is costly, involving as it does large preliminary works and the construction of wheel pits and

tunnels. At Niagara these were bored through solid rock at great expense. Another mode of developing water-power, some splendid examples of which may be seen in Northern Italy, is by the construction of canals from one point on a river, to another lower down, between which points the river has fallen, so that there is a difference of

Section of Weir.

FIG. 39.

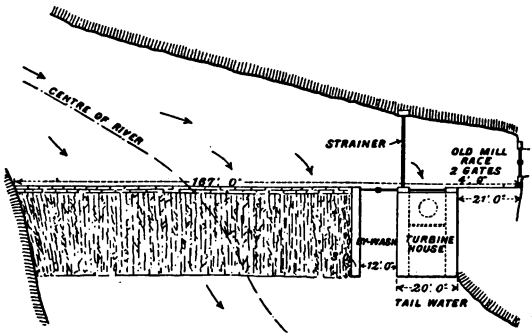


FIG. 40.—Plan of Weir.

elevation between the canal and river sufficient for operating water wheels. Then again, there is the lake high up on a mountain side from which the water is led out by a long pipe to the place of utilisation hundreds of feet below, or, if there be no natural lake, an artificial one may be produced by throwing a dam across a valley.

Each of these general plans necessitates special knowledge, some of which is quite outside the scope of this

work, and if it were extended to include them it would become a work on civil engineering. It is nevertheless impossible to avoid touching upon some of the allied subjects which are included under the comprehensive subject. As the smaller water-powers will become increasingly valuable in view of the possibility of utilising them through the medium of electricity, it is best to represent them by a description of a small installation which works under a head of about 6 ft. and from which some 50 h.-p. is obtained. This installation upon the Newry River at a point just above the town is built upon the site of an old dam, a section of which is shown in Fig. 39. It will be seen from the plan (Fig. 40) that the watercourse makes an angle where the dam is thrown across the stream. This dam is 167 ft. from bank to bank and of uniform cross-section, and before the water was drawn off for the turbine it passed over the top along the entire width. As shown in the section, the vertical up-stream face of the dam is made of rough masonry laid and pointed with lime mortar, and the long sloping back by boulders embedded in stiff clay.

The problem to be solved in this case was to ascertain the capacity of the proposed installation, for the flow of water was very variable owing to the rapidity with which the river responded to a downpour of rain, and it was important to fix the capacity of the plant at such a figure that the maximum output would be as large as could be maintained without incurring a wasteful efficiency by operating for part of the time at greatly reduced load. By measuring the flow over the weir and applying the Francis formula the discharge was calculated for heights varying

from 1 in. up to 30 in. and the results are shown plotted in curve A (Fig. 41). Thus for a height of 10 in. over the weir the discharge per foot of length would be about 150 cu. ft. per minute. The horse-power was not proportional to the discharge owing to a sharp turn in the river together with a restricted channel. It was observed that when the depth of water upon the weir was 3 in. the actual fall was 6 ft., and during an exceptionally heavy flood when there was 18 in. on the weir, the head dropped to 3 ft. From these data the horse-power was calculated for the varying depths on the weir and the results are shown on curve B. The equation for the curve was first obtained from which it was plotted (see Appendix B.). The ordinates give the horse-power for inches of weir depth measured as abscissæ. The actual available power is obtained by multiplying the value shown by the curve by the length of the weir in feet.

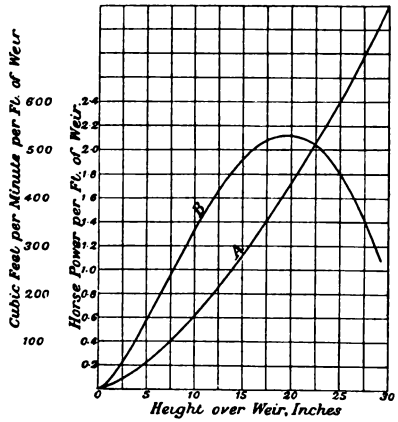


FIG. 41.—Flow over weir and horse-power.

As the flow corresponding to a maximum power would only be obtained by occasional heavy floods of short duration, the capacity of the plant had to be far short of this; and from records and observations of the flow at different seasons, it seemed feasible to operate a 50 h.-p. wheel for the greater part of the year, at or near full load. It was

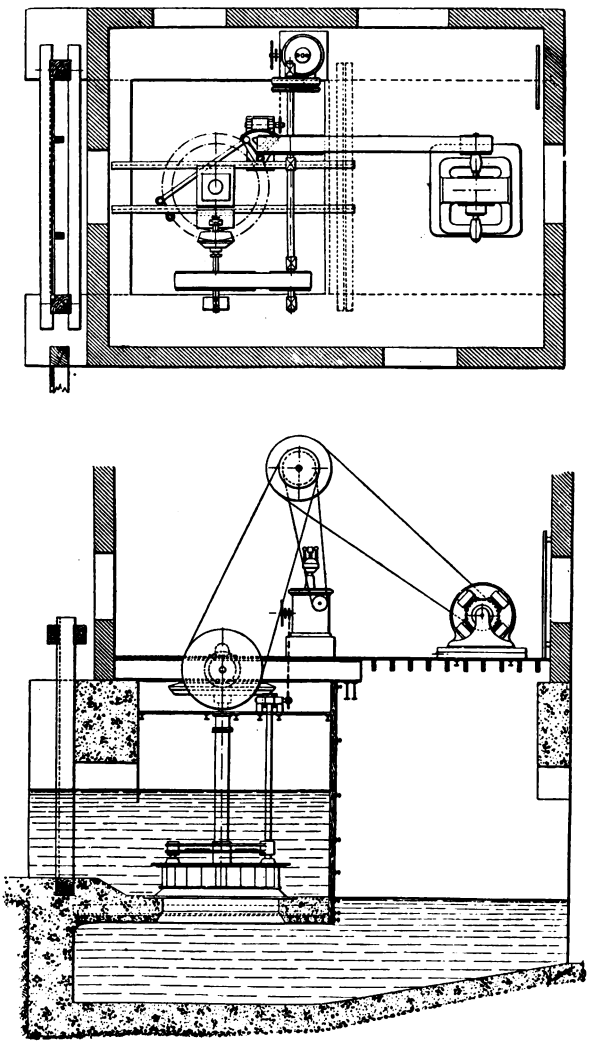


FIG. 42.—Plan and elevation of small turbine installation.

therefore decided to put in a turbine of 50 brake h.-p. with an hydraulic efficiency of 76 per cent., thus utilising at full load a supply of 6,200 cu. ft. of water per minute under an effective head of 5 ft. 6. in. (It was furthermore stipulated that the efficiency should be well maintained throughout a wide variation in the effective head.) Allowing 10 per cent. for losses in gearing between the turbine and generator, the actual horse-power delivered at the armature-shaft would exceed 44, so that a generator of 30 kw. capacity, with an efficiency of 90 per cent., could be driven at full load.

The turbine pit was excavated out of the weir, and a by-wash 12 ft. wide was provided so that when the wheel was not at work the water could be diverted by means of the gates on the up-stream side. The pit is flanked by two concrete walls which are carried up to a height of several feet above the level of the water on the up-stream side of the dam. The turbine is supported upon a floor of concrete reinforced by steel as shown in the sectional elevation (Fig. 42), and the back of the head race is constructed of 4 in. planking, tongued and grooved, the seams being caulked with oakum and served with tar and supported against the hydrostatic pressure by rails laid across and embedded in the side walls. The weight of the turbine is carried from the bearing above the level of the water and this bearing is supported by two 15 in. I joists placed across the wheel chamber at the floor level. By this mode of suspension access may be had to the bearing for lubrication and examination. The turbine is a mixed flow wheel of the kind generally employed for low heads.

The speed of this turbine is 55 revolutions per minute,

and the horizontal shaft revolves at three times this speed, this being the ratio of the bevel gear and pinion. From this shaft an overhead countershaft is driven at 300 revolutions per minute, and the dynamo is driven off this shaft by a belt, the normal speed of the machine being 650 revolutions per minute. The output of the dynamo is 130 amperes at 230 volts, and it is a compound-wound four-pole machine with an overload capacity of 25 per cent.

The foundations of the turbine house are carried down to solid rock at the tail race end of the house, and the concrete used was made up of one part of Portland cement to two parts of sand and four of granite chips, the largest of which would pass through a 6 in. ring. To decrease the cost of the work without materially diminishing the strength of the foundations, large boulders were embedded in the concrete, and the mixture was well rammed about them.

Due consideration was given by the author to a storage battery system by which the voltage of a supply current for lighting and power could be kept constant regardless of the condition of the river and the load; but it ultimately was decided to supply the line direct from the dynamo, and to provide a governor for maintaining a constant speed of the machine under all conditions. The governor, which is shown on the illustrations, and which is elsewhere described, is driven from the overhead counter-shaft, and the regulation is effected through a chain drive which operates a worm and sector, and the guide vanes of the turbine are rotated by this means. This type of governor requires occasional attention and must be kept supplied with oil, but when set it is entirely automatic, and the plant may be left throughout the day without an attendant.

A complete specification for a low fall vertical shaft turbine would be as follows:—

Specification for one 50 Brake Horse-Power Turbine for a Hydro-electric Plant.

1. *Size.*—The turbine shall be capable of developing 50 b.h.-p. at full gate when operating under an effective head of 5 ft. 6 in., with a flow of 6,200 cu. ft. of water per minute. This corresponds to an available power of 64·5 h.-p., so that the efficiency of the wheel at full gate must be 77·5 per cent. to develop the stated power. The guaranteed efficiency of the wheel at three-quarters and half load, as well as at full load, must be stated.

2. *Type.*—The turbine may be either a mixed or radial flow wheel with vertical shaft. The step-bearing which carries the weight of the wheel may either be located above the head water or in the tail race, but must be accessible for repairs or adjustment if placed in the latter position.

3. *Speed.*—The speed at full load should preferably not exceed 65 revolutions per minute; but it is not considered advisable that the manufacturer should modify the design of a standard wheel in order to come below this limit, and any reasonable departure therefrom will receive due consideration.

4. *Details.*—In addition to the wheel, guide-wheel, casing, base-plate and shaft, the contractor is to furnish all the iron work necessary for the support of the turbine in the pit. This would consist of steel I joists let into the wall on each side, with cast iron distance pieces to ensure a rigid construction. The beams are to be drilled for all bolts, and the length of the beams will be determined by

the width of the sluice-way, which may be taken as twice the diameter of the turbine casing. Sufficient length must be allowed for a good bearing on the wall, and the beams are to rest upon square pieces of ¼ in. iron plate which are also to be furnished by the contractor.

5. *Gearing*—The vertical shaft is to be provided with a mortice bevel wheel, made in halves, bored, and with keys fitted. A machine-moulded cast iron pinion to mesh with the gear is to be provided, but this will be fitted to a horizontal 3½ in. shaft supplied by the purchaser. The pedestals and bearings for the horizontal shaft will be also supplied by the purchaser, but the contractor is to furnish the cast iron yoke and bearing for the top of the vertical shaft. This vertical steel shaft will be approximately 10 ft. long, the exact length to be determined later. The gear ratio is to be 3 to 1, the diameter of the gear to be stated in the tender, but which is not to exceed 5 ft.

In addition to everything necessary for the proper operation of the turbine, a set of spare spanners is to be provided, together with a spare set of bushings for the main bearings.

6. *Governor.*—As the turbine is to operate a constant potential electric generator, it is essential that an automatic governor be provided, which is to be guaranteed by the manufacturer to control the speed of the turbine within the following limits consequent upon the corresponding percentage changes of load here stated:

Sudden variations of load of 25 per cent.	4 per cent.
,, ,, ,, 50 per cent.	6 per cent.
From no load to full load . . .	10 per cent.

The contractor is to state where such a type of governor

as he proposes to furnish is in actual operation in connection with turbines of less than 100 h.-p., operating under a head not exceeding 6 ft., the successful operation of water-wheel governors under high heads being no criterion as to their reliability with such a small head as we have in this case.

7. *Hand Control.*—The requisite mechanism for controlling the turbine by hand is to be provided and is to be arranged for shutting down the turbine without the necessity of lowering the sluice-gate at the mouth of the wheel pit.

8. *Materials and Finish.*—The cast iron used in the construction of the turbine is to be of a hard, fine-grained quality, free from blow-holes, and the turbine vanes and guide vanes are to present a smooth surface to the water, and may be of steel cast into the wheel. All parts of the governing and controlling mechanism situated on the dynamo floor are to be well finished and neat in appearance, and the working parts are to be provided with oil cups of large capacity so that frequent attention may be unnecessary.

9. *Painting.*—All parts of the turbine and supports which are exposed to the action of the water are to receive two coats of anti-corrosive paint, and the governing mechanism and other parts in the dynamo room are to be painted black, striped, and varnished.

10. *Drawings.*—The contractor is to supply dimensioned drawings of the turbine he proposes to furnish, showing all overall and important dimensions from which the plan of the wheel pit and station may be prepared by the engineer.

Turbines are worked satisfactorily under lower heads than 6 ft. There is a Jonval turbine in Worcestershire for driving a mill which operates under 2 ft. head, and at

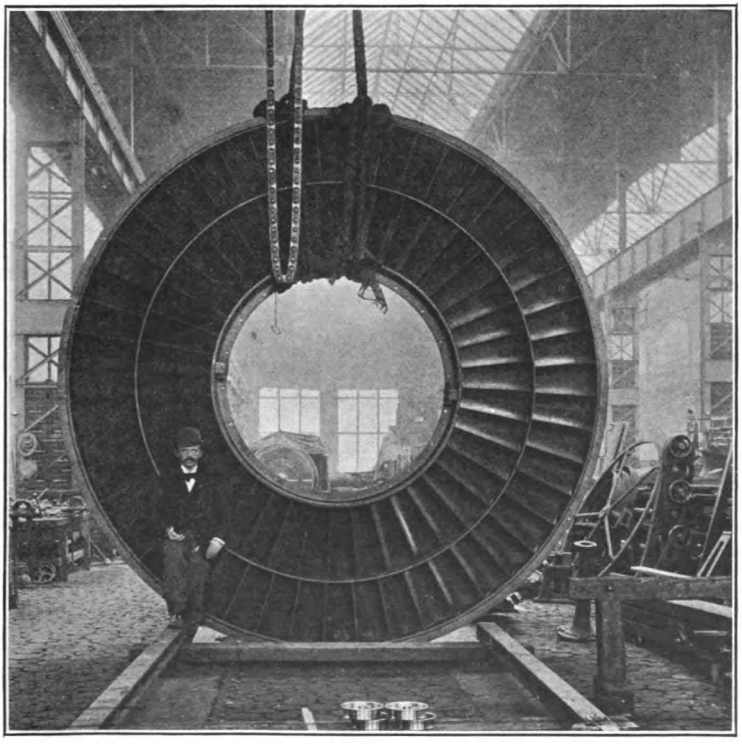


FIG. 43.—Jonval turbine working under a head of 2 feet.

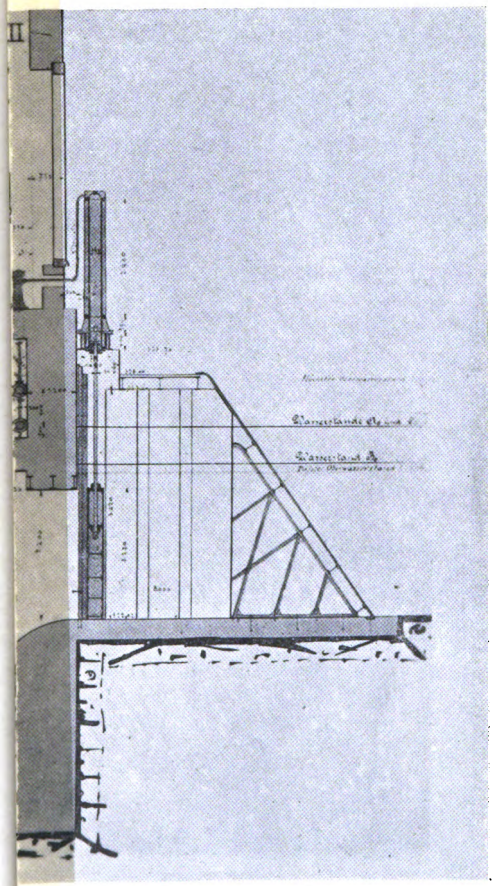
certain seasons of the year this is even reduced. The turbine is a parallel flow wheel with a vertical shaft, the diameter being 13 ft. 2 ins. (Fig. 43). It is made up of two concentric rings of vanes, and the inner set may be

closed to the water by a system of covers so as to limit the operation of the wheel to the outer ring.

At some periods of the year a head of 3 ft. acts upon the wheel and the 40 h.-p. required is got from the outer ring alone; but when the head is only 2 ft. both rings are used. This wheel makes 14 revolutions per minute, and passes about 14,000 cubic feet of water in that time. A very small head of water may thus be turned to profitable account where there is a good discharge. The cost of a turbine of this kind, including shafting, gearing, and sluice gates would be approximately £30 per horse power, or £1,200 for a 40 b.h.-p. wheel. The cost of foundations and special works would have to be added to this, but as it depends upon local conditions it is impossible to assign a value applicable to all cases.

The large gear upon the shaft of the turbine is 10 ft. 6 in. diameter, and has 132 cast iron teeth of 10 in. face. It meshes with a pinion 3 ft. 6 in. diameter, with 44 teeth, giving to the countershaft a speed of 42 revolutions per minute. This installation was designed by Mr. Alph. Stieger, M.Inst.C.E., and is a notable instance of a very low fall being successfully employed. There are many rivers upon which, at comparatively slight expense, a low fall might be utilised by such a wheel as this. The Jonval type is specially adapted for such a purpose, as the comparatively narrow wheel enables it to be used, even where the fall is very low.

An arrangement for driving an electric generator by three turbines on a vertical shaft is shown in Fig. 44. The water is admitted from the head race shown on the right, and the discharge from the wheels passes off through two



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channels. If the water is low the uppermost wheel is put out of action, while the two lower wheels are still working efficiently. This arrangement of wheels is not uncommon in Switzerland. The gate at the mouth of the turbine pit shuts the water off entirely, so that access may be had to all the wheels when desirable.

Turbines with horizontal shafts are coming into favour, especially for electric driving. Such a unit is illustrated in Fig. 31, p. 104. There is an advantage in having the electric generator upon a horizontal shaft as thrust bearings are dispensed with, and the generator and turbine are placed on the same floor with short rods and links for the governing mechanism.

The impulse turbine usually takes the form of the Pelton wheel for very high heads, some interesting examples of which are worthy of note.

To Switzerland at present belongs the distinction of utilising the highest head of water for driving water wheels. This installation is at Lake Tanay, a mountain lake of about 7·5 sq. kms. surface, out of which the water is conducted by a pipe line down the mountain side, and is discharged at a point 950 metres (3,115 ft.) below the surface of the lake. Of this great fall 920 metres (3,018 ft.) is the effective head acting upon the wheels, which yield an efficiency of 75 per cent. at full load and which, for every litre of water per second, do work at the rate of 9·2 h.-p. This power is employed in generating monophase electric current at 5,000 volts for distribution in the district of Vouvry, the generators being direct coupled to the Pelton wheels. The water is drawn off from the lake through tunnels, from which it is conducted down to the power

house through pipes. The first section of this pipe has a diameter of 500 mm., and is 635 metres long, the thickness of the metal varying from 7 to 11·5 mm. At the lower end there is a junction of forged steel from which two pipes of 340 mm. diameter convey the water the rest of the way. The water issues from the nozzles at a velocity of more than 100 metres per second and impinges directly upon the vanes of the wheels. At this high velocity it would be extremely dangerous to interrupt or check the flow except very gradually, consequently the governing of these wheels offers a problem of special interest, for while the working of the alternators demands close regulation, the safety of the pipe line is only ensured when the rate of change in the flow of water is kept low. It is pointed out in the next chapter how a destructive stress may be set up by checking the flow in a pipe line too rapidly, and the closing of the valve by hand in the ordinary way at the bottom of a pipe line such as this would be accompanied by disaster unless the pipe was made unduly heavy. The pipes in this installation are of welded open-hearth steel, and longitudinal joints are avoided. Such pipes have shown under test that they develop 100 per cent. of the strength of the steel, but it is customary to allow 80 per cent. only in making the calculations for thickness of material. The sections are joined by flange joints bolted together, and at the lowest point where the pipe has to withstand the greatest pressure the thickness is 18 mm. In order that the pressure due to the retardation of the mass of water in the pipe upon closing the valve should not exceed 5 kg. per sq. cm. (71 lbs. per sq. in.), it was made impossible for the attendant to close the valve in less than five seconds.

To each Pelton wheel there are two nozzles, one of which is fixed and the other is automatically regulated by a centrifugal governor. The jets from the nozzles have a cross section of 2 sq. cm. each, that of the fixed nozzle being circular, and of the adjustable, square. With the full load of 500 h.-p. the wheel has a speed of 1,000 revolutions per minute under the action of both jets. The wheel has a diameter of 1·20 m., and should the governor fail to act and the speed rise to the maximum of 2,000 revolutions per minute, the centrifugal stresses would still be well within the tensile strength of the material. The wheel consists of a disc to the periphery of which the vanes are attached, alternately right and left.

It is unquestionably true that wheels operating under such high heads are much less efficient than at moderate heads. The losses are due to friction in the pipes and to the fact that the water has a comparatively high residual velocity upon leaving the wheel buckets, and thus the kinetic energy is only partially used in useful work. Another interesting plant of the same kind is that of the Pikes Peak Hydro-Electric Co., at Manitou, near Colorado Springs. Pelton wheels are also used in this installation, the total head being 737 m. (2,417 ft.) which is the next highest head utilised to that at Lake Tanay. The entire length of the pipe line is 1,455 m., part of which consists of a light pipe on a small gradient, and the rest of a heavy pressure pipe 534 mm. internal diameter. Deducting the loss of head due to friction in the pipe, the effective head acting on the wheels is 671 m., and the pressure to which the pipe is subjected may therefore rise to about 1,000 lbs. per sq. in., to resist which it is made of $\frac{3}{4}$ in. steel plate with

seams of double-butt straps. An attempt was made to caulk the joints in this pipe with lead, but it was squeezed out in a thin film. Eventually an alloy of tin and lead was found to withstand quite well. The water strikes the wheel buckets with a velocity of more than 91 m. per second (204 miles per hour), and the governing is effected by deflecting

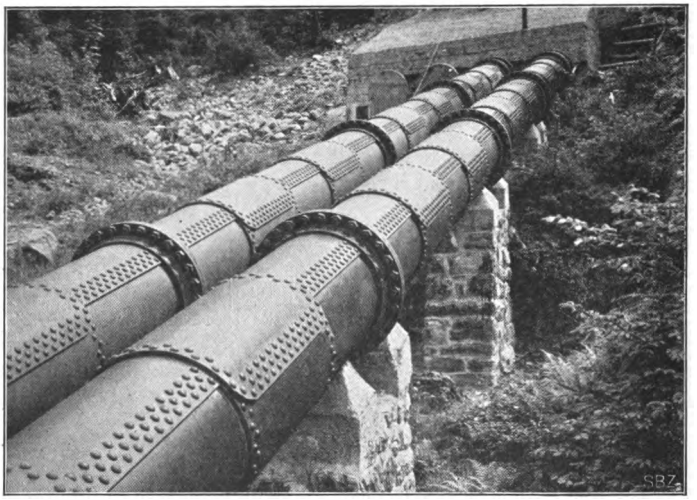


FIG. 45.—Riveted steel pipe lines for high pressure.

the jet, the nozzle being hinged for the purpose. As the impact of the jet would quickly wear through hard substances, the momentum is destroyed by directing the jet into a pool, the tail race having been so designed as to form a pool 40 ft. long through which the fierce jet expends energy, and terminates upon a cast-iron baffle plate at the end The valves are fitted with a slow worm and wheel attachment driven by an electric motor, so that it is outside

the power of the attendant to close them in less than 25 minutes. The steel pipe which is employed for installations of this kind is shown in Fig. 45, though welded pipe is now being largely used instead of riveted. A riveted joint cannot be relied upon for more than 80 per cent. of the strength of the solid plate.

The pipes are sometimes laid in trenches, and in many cases are surrounded by concrete. In others they are sustained in position by being supported upon piers of brick or stone, the flanges at the ends of the pipes being spanned by the piers, as shown in the illustration. An expansion joint or sleeve is necessary to prevent distortion and leakage at the joints, and is usually made in the form of a telescopic gland which allows longitudinal play.

Friction in Pipe Lines.

In every pipe carrying flowing water there is energy lost in overcoming the friction of the water against the walls of the pipe, and the energy so lost varies with the character of the interior surface of the pipe. Thus for rough cast iron pipes it is greater than for steel, for riveted pipes also it is greater than for welded. Then again, as a pipe in service becomes incrusted and corroded, the friction is greatly increased and consequently the loss of energy also becomes more serious with the life of the pipe. In choosing the size of pipe necessary to effect a given discharge, or to obtain the loss of head due to friction in a pipe carrying water at a known velocity, formulæ are used in which the constants have been determined by experiment. The most reliable series of experiments are those of Darcy, which were undertaken for the French Government, and by which

the value of the constants was found. The formula which is perhaps most employed by engineers is that known as the Chezy formulæ, by which the loss of head, h, in a pipe of length l, and diameter d, may be obtained, when the water is flowing through it with a given velocity. If the velocity v is in feet per second, and all dimensions are in feet, the loss of head h is :—

(1) For clean pipes, $h = 0{\cdot}0004 \times \left(\frac{l}{d}\right) \times v^2$.

(2) For incrusted pipes, $h = 0{\cdot}0008 \times \left(\frac{l}{d}\right) \times v^2$.

If v is in metres per second and all other dimensions are in metres, the loss of head h is :—

(3) For clean pipes, $h = 0{\cdot}0013 \left(\frac{l}{d}\right) v^2$.

(4) For incrusted pipes, $h = 0{\cdot}0026 \left(\frac{l}{d}\right) v^2$.

The loss of head for incrusted pipes is taken to be twice that for clean pipes, the coefficient in equation (2) being double that in (1). These formulæ, though not strictly correct for all velocities and all sizes of pipe, may be taken to be sufficiently accurate for the moderate velocities usual in pipes and penstocks, and for pipes from four inches in diameter upwards.

If the pipe be riveted, and therefore offering greater resistance to the water, formulæ (2) and (4) ought to be used, and it is sometimes expedient to calculate the possible loss of head for a new pipe by using the formula for incrusted pipes so as to anticipate the inevitable incrustation and clogging of a pipe in service. As an example of the use of the foregoing equations, suppose it is desired to obtain the

loss of head occurring in 500 feet of 8 inch cast iron asphalted pipe when the velocity of the water is 4·3 feet per second,

$$h = 0{\cdot}0004 \times \left(\frac{500}{0{\cdot}67}\right) \times (4{\cdot}3)^2 = 5{\cdot}52 \text{ ft.}$$

The highest head in use in England at the present time is employed to drive a Pelton wheel at the Croeser slate quarries in Wales. This installation formed the subject of a recent paper,[1] in which the author states that two natural lakes discharge into the Cwmfoel valley, and by building a dam across the valley about 12 acres of water have been impounded at an elevation of 860 ft. above the power house. The higher lakes may be used to supplement the storage of the reservoir, or may be applied directly to the wheels in the power house. The steel pipe line is 3,200 ft. long, and it conveys water to two wheels, one of 375 b.-h.-p., and the other of 25 b.-h.-p. To avoid dangerous water hammer, the pipe line is provided with an air-vessel of 30 cu. ft. capacity. The hydrostatic pressure being more than 25 atmospheres (367 lbs. per sq. in.), it was necessary to fill this vessel with air at the same pressure. Under a head of 861 ft. with a flow of 276 cu. ft. per minute, and a peripheral speed of 40·7 per cent. of the theoretical velocity due to the head, the wheel is stated to have attained an efficiency of 87 per cent., which would place it far above any other type of prime mover as an efficient machine if all the losses have been properly charged up against it. As such efficiencies have been claimed by others for these wheels there appears to be some ground for supposing that, at the best velocity,

[1] The Institution of Civil Engineers. "The Application of Hydro-electric power to Slate-mining," by Moses Kellow, Assoc.M.Inst.C.E.

and with a steady load, this efficiency may be momentarily realised, but if the wheel is working under the conditions that practice implies from day to day, the efficiency would fall short of this.

The Cost of Water Power.

The consideration most generally affecting the choice of power for a given purpose is the cost, both first cost or capital charge, and running costs or maintenance. When a supply of water from which power may be obtained is available, the choice may therefore rest between water-power on the one hand and power derived from fuel either through the steam or internal combustion engine on the other. As the influence of "convenience" cannot readily be measured, even for a specific case, and still less for a general comparison, the problem, as far as we are concerned at present, resolves itself into a question of relative cost between the two forms of power, as to which should be used where both are available. But there is an essential difference between an estimate for a steam plant and for a water-power installation. The former is but little different for plants of given power laid out at different places accessible by rail, while an estimate for the development of a water-power for one locality would be entirely wrong for another. Local conditions affect the one but slightly, while with the other they affect the question so seriously as to make the issue dependent upon the result of a careful study of the locality. It is not to be assumed that the cost of installation of steam plants is quite independent of the locality, for often heavy ground rents, high charges for condensing water, taxes, and numerous restrictions and observances

which municipalities impose upon a power company, render the power-supply close to a large market somewhat more costly than it would be if the power station were situated in a country district, but the machinery and buildings would cost the same, irrespective of the locality, and it is with the cost of these that we wish to compare the cost of the works generally necessary for developing a water-power. These works comprise the excavation of turbine pits, digging canals or laying pipes and penstocks, building dams and deepening channels, in short all the work necessary before the turbine can be started, which may include any or all of the foregoing processes. Then there is the cost of the turbine and accessories, such as the gearing and devices for supporting it in the pit, with the thrust bearing and countershaft for taking off the power, and finally the cost of the building. As against these items the steam plant comprises a boiler plant with coal hoppers, and possibly mechanical stokers and ash conveyors, steam engines with the piping, condensers, air pumps, and coolers, and possibly a feed water heater, hot well, and also a steam superheater. The variety of appliances and machinery going to make up a large steam plant is legion, but for small installations, with which it is intended to compare water-power plants of similar sizes, many of these auxiliaries disappear, and some are replaced by manual labour. Then there are the heavy foundations for the reciprocating engine to damp out vibration, and the building for protecting the machinery from the weather. Steam turbine plants effect a saving in foundation costs, but as this substitute for the reciprocating engine is chiefly economical in units of large size, it is only in the

consideration of large undertakings that it invites comparison.

Engine foundations and buildings have their counterpart in the water-power installation in the works necessary for conveying water to the wheel, also the buildings or turbine house. The cost of the former may be directly ascertained, as the quantity of concrete or brick work to put down for an engine of given size is known, and can be obtained from the makers in the light of their experience. The same is not true for turbine foundations or pits, the design and size of which are dependent upon several conditions prevailing at the site. In some cases necessary precautions against floods necessitate heavy construction with foundations carried far below the bed of a river until a firm foundation is reached, and the cost of construction increases accordingly, especially when the work has to be kept dry by pumping, or is in danger of being washed away by rises in the river. Then again the extraneous works often necessary to obtain the requisite fall upon a river add greatly to the first cost of water-power installations. Such for instance would be a canal which opens out from a river at a point above the site of the plant and conveys the water to the turbine under a head equal to the difference in elevation between the water in the canal and that in the river at the point in question. Some such canals, which form essential features of many of the beautiful installations in Northern Italy, and which convey water for several kilometres, have been so costly as to be quite prohibitive in coal mining countries where cheap steam coal may be obtained. In fact the writer ascertained that the interest on the first cost of one famous installation added to the cost of

maintenance made the cost of power almost equal to that derived from high priced fuel in a large city to which the power was conveyed by transmission lines. True, the losses in transmission, amounting to 8 per cent., told against the water-power plant, but notwithstanding this, the difference in cost of a horse-power delivered from the two sources was barely sufficient to justify the enormous expense of a canal cut through rock for several kilometres. And so it is in many other instances where, at a great outlay of capital, water power is delivered in competition with heat-engine power with very little difference in cost, owing to the heavy capital cost of the works necessary to develop it, though the running costs and maintenance charges may be extremely low compared with steam or gas power. The development of such water powers remote from the centre of distribution is a problem of a special kind, while the use of small powers located close to a manufacturing centre and in direct competition with cheap steam power is quite another. Without prejudging the question as to the distance it is commercially practicable to transmit power (the maximum distance at present being 232 miles from the De Sabla power-house of the California Gas and Electric Co. to Sausalito at 50,000 volts), the limitations to which are chiefly in the cost of the copper line, and the difficulties incident to the insulation of the line at very high potentials, it is clear that there is a limit which may be nearer than some of the extreme advocates of long distance transmission imagine. The price of copper tends to rise (it is now £118 per ton); there is yet no satisfactory means of insulating a line for pressures greater than 60,000 volts without enormous loss at the points of support; the high

tension direct current system is still in a tentative stage, so that much still remains to be done before a prophecy as to the maximum economic distance is worth anything. It is true that in some cases a heavy loss in transmission may be accepted, so that only 25 or 30 per cent. of the hydraulic power is available at the distribution end, and still be able to successfully compete with steam power, as the water may cost nothing. This argument may however be gainsaid by a glance at the other side of the balance sheet showing the interest on first cost of a large plant, the cost of maintenance of a long pole line, with a continual liability to interruption by the weather or by the acts of vandals, followed by serious charges against the power company for failure in the supply. As long distance transmission schemes vary so much in character and involve such a variety of special interests and commercial considerations, it is not possible within the limits of this little work to take up this great branch of the subject which the author hopes to enlarge upon in another place. For the present, therefore, to simplify the problem, we will omit the pole line and the electrical side of hydro-electric installations and confine our attention to the case of water power versus steam or gas power delivered at the shaft of a generator or other machine which is to be driven.

The cost of a turbine and accessories necessarily varies with the power it is intended to develop, but also with the type of wheel, the head under which it is to operate, and the speed. As the speed or velocity of the buckets or vanes is a direct function of the head (in the impulse turbine being approximately one half that due to the free fall of water), this may be attained by wheels of various sizes with

correspondingly varying angular velocities, the revolutions being inversely proportional to the diameter for a given speed of vane. But the cost of the wheel, accessories, and setting increases with the diameter, so that a fast running wheel for a given output under a stated head is cheaper than one with a lower angular velocity. The diameter,

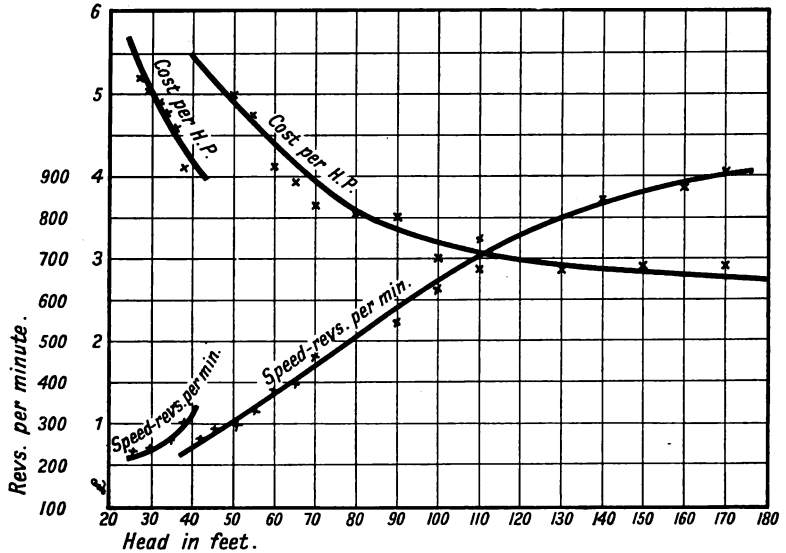


FIG. 46.—Relation between the cost of a turbine and the head of water.

however, must in all cases be sufficiently large to allow for the passage of the water without choking. As the speed depends upon the head it follows that the cost of a turbine decreases as the head increases. This is shown by the curve in Fig. 46, which is plotted from a price list taken from a manufacturer's catalogue, and which refers to a vortex turbine with horizontal shaft complete with accessories.

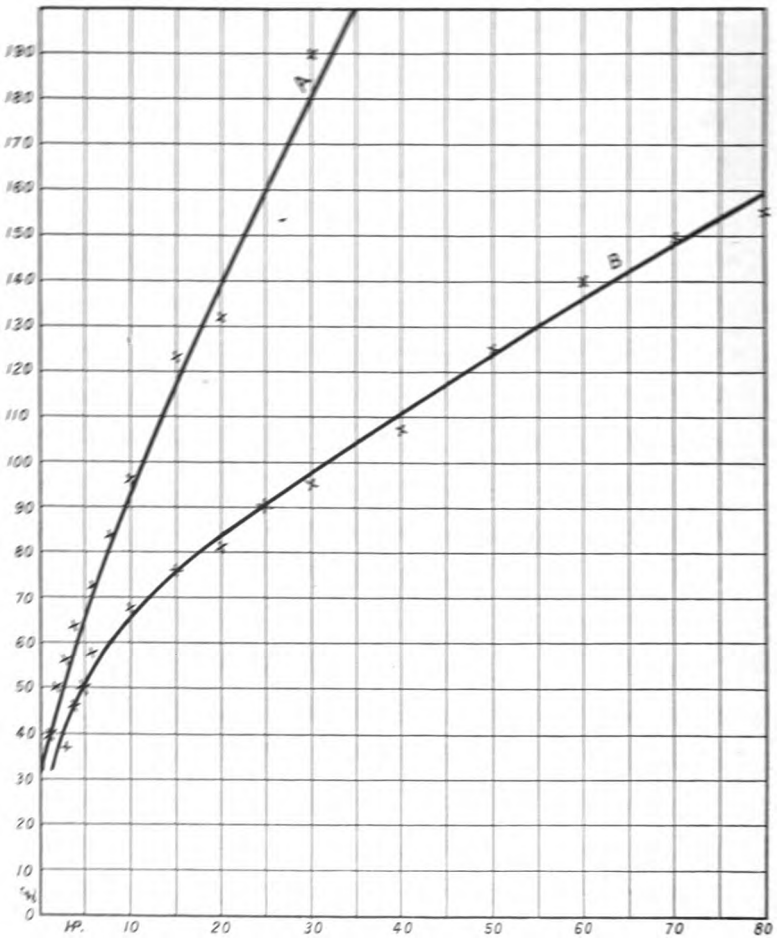


FIG. 47.—Relation between cost of turbines and rated horse-power.

The ordinates are pounds sterling *per horse-power* and are obtained directly from the list by dividing the cost of the turbine in pounds for a given head by 40, which is the rated horse-power of the wheel chosen. The short curve refers to another type of wheel, by which it will be apparent that the cost per horse-power decreases more rapidly with increase in head than with the other type of wheel, which is plotted for a range of head of 40 to 180 ft. The points represent the actual costs *per horse-power*, and the smooth curve drawn through them is designed to indicate the way in which the cost changes with the head. For instance, at 50 ft. head the cost of the wheel is £4·9 per h.-p., or, for the 40 h.-p., £196. At 140 ft. head 40 h.-p. is obtained from a turbine costing approximately £2·9 per h.-p., or £116 for the wheel. This decrease in cost per h.-p. is due to the fact that under an increased head a smaller volume of water is passed to develop the rated power, and the size of the wheel may therefore be reduced. The speed curves on the same sheet show that the speed of the wheels is not directly proportional to the head, for if it were the curve would be replaced on the diagram by a straight line. For the highest heads the speed is less than it should be if it were always proportional to the head. Fig. 47 is put in to show how the price of turbines of this type varies with the rated horse-power of the wheel. The curve A refers to wheels working under 24 ft. fall, and B is for the same type of wheel under a 90 ft. fall. The abscissæ are the rated horse-power of the wheels and the ordinates are pounds sterling. Taking the two curves it is seen at once that the curve B bends over more sharply than A, and is inclined at a lesser angle to the horizontal axis. This shows the reduced cost of a wheel of given

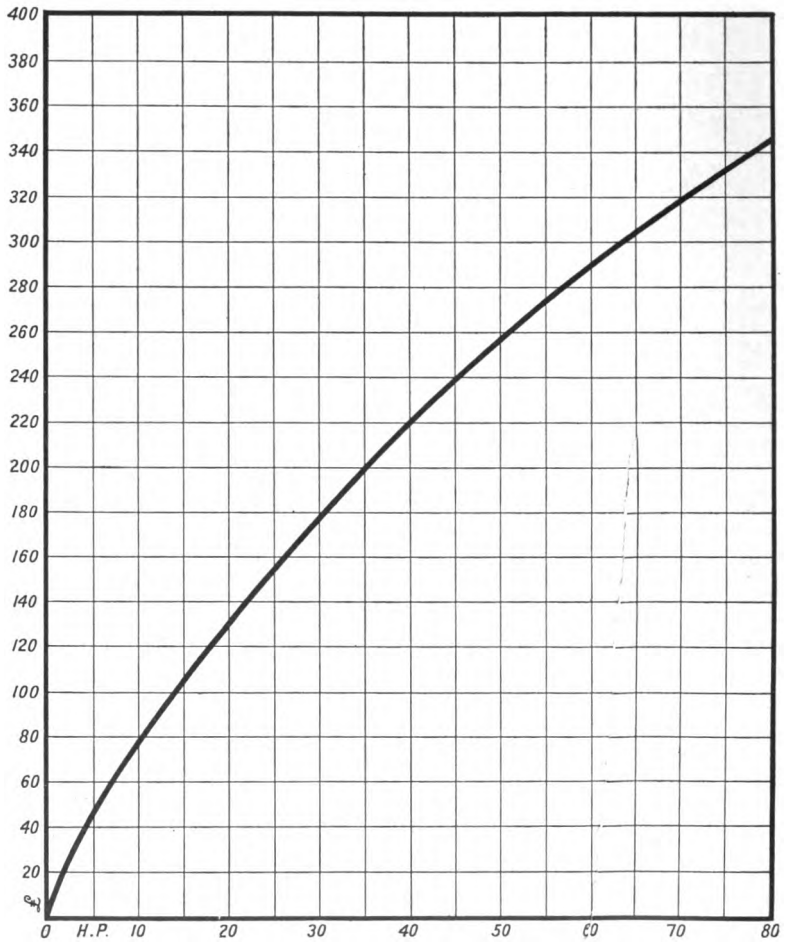


FIG. 48.—Relation between cost of 500 volt direct current generators and size of machines.

power working under a higher head, and also that the capital cost per horse-power is considerably less.[1] There is no difference in this respect between turbines and other kinds of machinery, for additional power costs less as the size of the machine or engine is increased, though it is perhaps more apparent in turbines than other classes of machinery. Fig. 48 is a curve showing the cost of slow speed electric generators per horse-power, the slope of which is again an indication of the way the *price per horse-power* varies with the capacity of the generator. The diminution of this figure as the size of the generator is increased is less rapid than for the hydraulic turbines. The prices of turbines taken from a single maker's catalogue are naturally of limited value to a purchaser, and are only introduced here to illustrate how the cost per horse-power is effected by the size of the unit, and are not to be taken as indicative of the state of the market for such machinery. The facts which they are intended to illustrate are, however, independent of the price of the machinery by any one maker or of any special design or type. The water-power plant is therefore no exception to the general rule that the cost

[1] These curves may be referred to equations which take the form $y = a\,x^n$. Accordingly the slope of the curve at any point is a direct indication of the *price per horse-power*, for this price is obtained by dividing y by x at any point, and as:—

$$\frac{d\,y}{d\,x} = a\,n\,x^{n-1} \text{ and } \frac{y}{x} = \frac{a\,x^n}{x} = a\,x^{n-1}$$

$$\text{therefore } \frac{y}{x} = \frac{1}{n} \times \frac{d\,y}{d\,x}.$$

As $\frac{d\,y}{d\,x}$ is the slope of the curve, the variation in *the price per horse-power* is therefore readily seen at a glance.

per h.-p. diminishes with the size of the unit, being greatest when the units are small.

The works necessary to develop a small water-power (say 50 h.-p.) comprise the excavation and construction of the turbine pit, the construction of the turbine house and the installation of the machinery, and in many cases the building of a dam to obtain the necessary fall. As, however, dams are not always a part of the work, they may be regarded as extraneous for the purposes of making comparisons. The capital cost of such works completed, with the turbine ready for running, would be, for falls up to 8 ft., about £1,200 to £1,400, according to the state of the soil as affecting the depth to which the foundations have to be carried. This total cost would be made up as follows :—

	£
Turbine, with gearing, draft tube, gates, and step bearing (£11 per h.-p.) . . .	550
Excavation of turbine pit and construction of arch or ferro-concrete support with water back, side walls, and turbine house . .	750
	£1,300

Total capital cost per h.-p., £26.

The cost of operation and maintenance per annum of such an installation would be as follows :—

	£	*s.*	*d.*
Interest on capital at 3½ per cent. . .	45	10	0
Depreciation at the rate of 4 per cent. (which allows for entire replacement of plant in 25 years)	52	0	0
Carried forward . . .	97	10	0

	£	s.	d.
Brought forward	97	10	0
Proportion of wages of attendant, whose time is divided between the hydraulic plant and the care of other machinery	50	0	0
Oil, waste, and other supplies and repairs .	20	0	0
Total cost for 50 brake h.-p. per annum .	167	10	0
Cost per horse-power per annum . .	3	7	0

If the plant be in operation for 12 hours per day, 300 days in the year, the cost *per horse-power hour* becomes 0·22*d.*

Compare this with the cost of operation of a steam plant of similar power consisting of a multitubular boiler set in brickwork with chimney, feed pump, etc. The engine to be of the slide valve pattern with throttle governor, and wide rim wheel for belt.

	£	s.	d.
Cost of plant on foundations with shelter	770	0	0
Cost per h.-p.	15	8	0

The total cost of operation would be as follows:—

	£	s.	d.
Interest on capital at 3½ per cent. .	26	18	0
Depreciation at 5 per cent. . .	38	10	0
Wages of attendant (entire time) .	78	0	0
Oil, waste, supplies, and repairs .	38	0	0
Coal at 10*s.* per ton for 3,600 hours (4 lbs. per horse-power per hour, including waste)	160	14	3
Total cost for 50 brake h.-p. per annum	£342	2	3

Cost *per horse-power hour* = 0·46*d.*

The steam plant is therefore twice as costly per horse-power as the water plant, and unless the difference is made up in charges for water, which are not allowed for in the estimate, the price of a water generated horse-power ought to be one half that of steam generated power for small plants. It may be said that a liberal allowance is made for coal consumption, which is put at 4 lbs. per horse-power hour, but this includes the wastage, and no deduction is made for ash and non-combustible material, which, in a cheap steam coal, may be a large proportion by weight.

The depreciation upon the water-power plant is based upon the entire cost, but as the machinery which is most liable to deterioration is only 42 per cent. of the whole capital cost, this allowance errs on the side of liberality. On the other hand, a boiler plant is subject to comparatively rapid deterioration, especially when fed with a low grade fuel, with perhaps a feed water charged with impurities which form a hard scale of carbonate or other salts of lime upon the shell. When it is remembered that a scale 1·5 mm. thick on the plates of a boiler causes a loss of one-eighth in efficiency, it will be seen that a neglected boiler quickly raises the expenditure on fuel also. The foundations and setting for the steam plant are a small proportion of the total cost as compared with the water-power installation.

A steam or gas engine plant is sometimes necessary as an auxiliary to a water-power plant during times of low water in dry weather. As the use of artificial light is generally less at seasons of low water, water-driven electric plants may often be maintained throughout the year without the aid of auxiliary sources of power by proportioning the plant to the

capacity of the winter lighting load. It will be observed from the above figures that the stand-by charges for a steam plant (*i.e.*, the cost of the plant in capital charges, interest, depreciation, etc., irrespective of whether it is running or idle), relative to the total cost, is a smaller proportion than for the water-power plant. Allowing half-time for attendance and lubrication, etc., in each case when the plants are shut down, the stand-by charges for steam are 38 per cent. of the whole cost of operation, and for water they are 79 per cent. The total cost of operation of the steam plant is, however, one half that of the water plant.

A gas engine, which consumes 20 cu. ft. of illuminating gas per brake horse-power per hour (and this figure may be taken as an average for engines up to 50 h.-p. using gas of the calorific quality of London gas), and which may be quickly brought into service at times of low water, has advantages over a steam engine where gas is available at a fair price. Taking the price at 2*s.* 6*d.* per thousand cubic feet, the cost of fuel alone for the gas engine works out at 0·6*d.* per horse-power per hour. The small gas producers will yield a supply of gas sufficient for a brake horse-power at a consumption of 1·5 lbs. of coal per hour. Indeed, some of the best forms of producer have brought down the consumption to less than one pound, but even taking 1·5 lbs., the gas engine with producer, fuel alone considered, is a cheaper method of producing power than the steam engine for small sizes, even though a better quality of coal is necessary to yield the gas. With large size units where compound engines or steam turbines are used with super-heated steam at high pressure and condensers, the consumption of

good coal has been brought down lower than one pound per horse-power per hour, but this has not been attained with small simple steam engines, and in this the gas producer and engine excels. The depreciation and maintenance of a producer plant bear comparison with that of steam boilers, though in general are somewhat greater.

CHAPTER VIII.

THE REGULATION OF TURBINES.

THE speed control of turbines has assumed an importance in recent years in consequence of the exacting requirements of the electrical engineer. While in every case where a turbine or other hydraulic motor is set to drive against a variable resistance some degree of speed regulation is desirable, if not actually necessary, the high degree of regulation necessary for dynamo driving has invited attention to the problems of regulation in general, and has resulted in successful achievement under most conditions. It is understood that the control to which reference is here made is that which is effected automatically by a mechanical arrangement designed to adjust the power supplied to the turbine by the water to the immediate necessities of the machinery to be driven, so that a constant speed of the machinery may be maintained at all times, whatever may be the fluctuations of the load on the one hand, or of the supply of water on the other.

There is but little to guide the engineer in the study of the regulation of water turbines by similar work accomplished for the steam engine. In the one case the working fluid is incompressible, possessing comparatively great inertia; in the other we have the opposite to these physical attributes, which greatly simplifies the problem and renders steam engine regulation possible without cumbrous apparatus

which we must perforce employ in hydraulic installations. The centrifugal governor, which is usually employed alike for both types of prime mover regulation, represents the extreme limit to which analogy would lead us. In the case of the steam engine the centrifugal governor, directly applied to the throttle or cut-off gear, has sufficient influence to overcome the inertia and frictional resistance of the valves; in the water turbine the movement of the governor is utilised indirectly in a relay to operate the heavy gates or controlling mechanism by which the flow of water is adjusted. Thus in the steam engine the flow of the highly elastic fluid is checked or increased by the movement of a delicate valve which is attached to the centrifugal governor and which moves therewith; but in the water turbine the acceleration or retardation of large volumes of water necessitates the application of forces at the controlling gates greater than could be got directly from a centrifugal governor of reasonable dimensions. In large marine engines a steam relay valve is sometimes employed, by which the attendant, operating upon the throttle valve of a small engine, controls the main valve which is too heavy to be conveniently manipulated directly. Supposing such a valve to be under the control of a centrifugal governor which would keep the speed of the engine constant by acting as a relay, we would have a parallel to the relay arrangement customary in water turbine installations. There is indeed no other plan by which regulation can be accomplished satisfactorily, so that a relay is an indispensable adjunct to the governing mechanism.

The sudden stoppage of a mass of water flowing in a pipe gives rise to a momentary pressure of great intensity.

The magnitude of this pressure above the normal pressure when the flow is uniform has been investigated by theory and determined to some extent by experiment. The theoretical investigation is somewhat too recondite to be entered into here; moreover assumptions have to be made in the mathematical treatment of the problem which would divest it of practical utility, were it not that experiment has to a reasonable extent supported the conclusions arrived at. By assuming an absolute rigidity in the material of which the pipe is composed, it has been shown that there are waves of pressure induced in a pipe by the interruption of the flow, and the maximum pressure may be determined.[1] If p (expressed in pounds per square inch) is the pressure induced by the sudden interruption of the flow *above* the normal pressure, and v is in feet per second, the equation for water is $p = 20\,v$. In metric units (metres per second and kilogrammes per square centimetre) this becomes $p = 4{\cdot}6\,v$. The instantaneous closing of the valve in the pipe is also assumed, but in practice such a rapid interruption of the flow would not occur, and therefore the pressure liable to be induced in the pipe would be considerably less than the above; but yet it has to be reckoned with in long lines of pipe, and methods have to be taken to guard against the possible destruction of the pipe from this cause. This may be effected by preventing water hammer altogether, such as by valves which cannot be rapidly closed, or by providing an air chamber on the pipe, or else by increasing the thickness of the pipe over that designed to withstand the hydrostatic pressure due to the head (see Appendix). Experiments carried out

[1] "Applied Mechanics," by Prof. J. Perry, p. 503.

by Mr. Latting on a pipe line 2,395 ft. long and 8 ins. in diameter with a 2 in. jet gave some interesting results. Under 302 ft. head, when the valve was closed in 25 seconds, the pressure of 107 lbs. in the pipe rose to 143 lbs. Theory gives 139 lbs. The inference to be drawn from these experiments is that this phenomenon stands in the way of the close regulation of water turbines, for this pressure comes upon the gates and valves and upsets the regulation.

For convenience in discussing the details of governing mechanism, hydraulic turbines may be divided into three classes according to the head under which they operate. The customary general distinction between Girard or impulse turbines on the one hand, and reaction or "drowned" wheels on the other, is not so relevant for the purpose as the head under which a wheel is designed to operate. The total range of head at present in use may therefore be arbitrarily divided into three parts, the division between one class and the next being necessarily obscure, but sufficiently definite for the purpose of allocating any turbine and designating it as :—

(1) A low fall; (2) a medium fall; and (3) a high-fall wheel.

Class.	Actual Range of Static Head in Feet (in Metres).	Corresponding Range of Pressure. Lb. per Sq. In. (Kg. per Sq. Cm.)
1. Low fall .	1.5 (0·457)—30 (9·15)	0·65 (0·046)—13 (0·915)
2. Medium fall .	30 (9·15)—300 (91·5)	13 (0·915)—130 (9·15)
3. High fall .	300 (91·5)—3018 (92·0)	130 (9·15)—1310 (92·3)

The mass of the movable parts of the regulating mechanism in turbines is very considerable, and the

friction opposing the motion is also great, as they are of necessity constructed to withstand rough usage and cannot always be lubricated efficiently, especially when submerged. This friction and inertia together offer a great resistance which must be overcome by a considerable application of force on the part of the governing mechanism, for a rapid acceleration is required for accurate governing.

The following four types of gate mechanism are employed for regulating the flow of water to turbines:—

(1) A circular iron or steel gate interposed between the guide vanes and wheel, which may be raised or lowered parallel with the axis of the turbine.

(2) Movable guide vanes capable of rotation through a small angle about a pivot near their geometrical centre or centre of gravity. (See Fig. 28, p. 98.)

(3) Needle valves or deflecting nozzles as used with impulse wheels acting under high heads, particularly with the Pelton tangential wheel.

In addition to these gates, which may be operated upon by the automatic regulating devices, there are also the usual forms of gate for closing down the plant or for shutting off the water preparatory to effecting repairs, but which are never connected with the automatic regulators.

(4) In turbines constructed on the partial admission principle the flow may be regulated by gates which control the number of the openings between the guide vanes.

The circular gate (1) is usually balanced by counterpoises so that the force necessary to raise or lower it is the same, and is that required to accelerate twice the mass of the gate in addition to overcoming frictional resistances. It may be noted that, to lift the gate at a given acceleration

without a counterpoise, a greater force is necessary than with a counterpoise when the required acceleration is less than that due to gravity. Otherwise a counterpoise, by doubling the mass to be moved, would require a greater force. If W be the weight of the gate and a the acceleration, the force required to accelerate it against gravity and without a counterpoise will be :—

$$F = \frac{W}{g} a + W.$$

With a counterpoise :—

$$F = \frac{2\,W}{g} a.$$

As the friction is substantially the same in both cases, when $a = g$ these two values of F are the same. The convenience of having a constant resistance for lowering and raising is such that a counterpoise is usually used. The Niagara turbines are equipped with gates of this kind, which, with their long connections to the regulators, weigh about 12 tons, all of which has to be put in motion every time the regulator operates (Fig. 29, p. 99). This form of gate has many advantages, and may be used with multi-section turbines, so that the position of the gate determines the number of rings of the turbine that are in operation at a given time. There is a loss of energy owing to eddies being formed round the edge of the gate, and also from leakage, as the clearance between guide vanes and wheel is necessarily large. The resistance offered by the water to the movement of this form of gate is small, for though submerged, the motion is edgewise and the pressures are balanced. It is not liable to become choked with impurities which may have passed the grating at the mouth of the

penstock, and there are no joints or bearings exposed to the action of the water which cannot be properly lubricated.

The employment of movable guide vanes (2) in turbines of the Francis type is now very general, especially for low heads where the velocity is small. The vanes are arranged to rotate through an angle of about 20°, and the water may be completely shut off from the wheel by turning them so that the openings between them are closed as they touch (Fig. 28, p. 98). The resistance offered to the movement of this ring of vanes is comparatively great, and in consequence they are generally actuated through a worm and wheel or a similar large reduction gear. The pivots upon which the vanes rotate are of steel or wrought iron, and are stepped into holes bushed with bronze with a loose running fit. The method of communicating the motion from the governor to the ring of vanes varies with different manufacturers. Some employ a steel ring with pins projecting therefrom in a direction parallel to the turbine shaft. These pins ride loosely in slots cut in the top of the guide vanes, and the rotation of the steel ring through a small angle by means of a sector and pinion is communicated to the guide vanes all round the circumference. Care must be exercised to prevent rubbish from being carried past the gratings when, with low falls, this form of gate arrangement is adopted. Stones and rubbish are liable to be carried along the bottom of the head race by the current, and, by entering between the vanes, would prevent regulation, if indeed serious damage was not done. To avoid this it is expedient to raise the turbine 3 ins. above the floor of the head race, and the rubbish may then be removed when the pit is drained

by the closure of the main gates, which should be done from time to time. The pivots upon which the vanes turn are not placed exactly at the geometrical centre, so that the water pressure results in a turning moment which has to be overcome for rotation in one direction, and which assists rotation in the other. A vane which is suspended in a moving fluid on pivots located eccentrically tends to place itself with the long side down stream. In practice the pivots are closer to the inner edges of the vanes than to the outer, which is necessary so that the variation in the clearance between vanes and wheel may be as small as possible with the rotation of the vane. An objection to this form of regulation has already been alluded to, viz., the loss of energy by shock owing to the variable angle of the guide vanes, for, with a certain speed of wheel, there can only be one angle at which the water enters tangentially, and therefore only one position in which there is no loss.

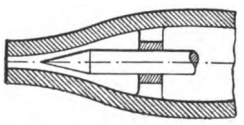


FIG. 49.—Needle nozzle.

The needle nozzle by which the stream of water for the Pelton wheel may be controlled is shown in Fig. 49. It consists of a nozzle, the interior contour of which is such that there is a minimum of dispersion in the jet as it issues. A spear is located axially inside the nozzle, and is capable of being moved longitudinally by the governor. When moved to the left the area of the passage for the water is diminished. For very high heads, where the interruption or partial suppression of flow by such an arrangement might be dangerous, deflection nozzles of various kinds are used. The nozzle is hinged in some cases and is moved by

the governor, so that with a diminished load on the wheel part of the stream is diverted past the buckets. This is, of course, a wasteful method, but, generally speaking, water is plentiful in places where the Pelton wheel is found to be of service. Another form of deflection nozzle as used by Messrs. Theodor Bell & Co. is shown in Fig. 50. The nozzle is rectangular in section, the upper side being formed by the piece E pivoted at A. This is moved by the governor attachment B so as to increase or diminish the area of the nozzle at the mouth. A secondary nozzle D is closed by a cap C, which also rotates about the pivot A, and is moved through the same angle as E by the action of the governor. As the two nozzles are the same width in a direction normal to the plane of the paper, a partial closure of the large nozzle is compensated for by an opening of the smaller, so that the spouting area is always a constant, and therefore the flow is steady through the pipe, while the wheel receives that fraction of it which is required to maintain the speed constant at all loads.

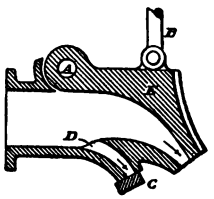


FIG 50.—Deflection nozzle.

The regulation of the Pelton wheel involves a curious hydraulic paradox which explains why it has been noticed that in some cases a diminution in the spouting area of the jet causes an acceleration of the wheel, and *vice versâ*. In other words, increasing the area of the nozzle involves a reduction in the kinetic energy of the jet, and closing it corresponds to an increase within certain limits. The explanation of this curious phenomenon was offered by

Professor Goodman,[1] who showed that it depended upon the loss of energy due to friction in a long pipe conveying water to the nozzle. If the friction be proportional to the square of the velocity in the pipe, and inversely to the diameter, any increase of area at the nozzle will, by augmenting the velocity, likewise augment the loss of kinetic energy by friction, so that the available kinetic energy of the jet may be diminished more than it is increased by the extra volume of water spouting from the jet of increased area. This balance between the loss of energy by friction on the one hand and increase on the other only occurs between narrow limits, and in cases where the pipe-line is long and the friction losses considerable. Should such conditions be favourable, a maximum power in the jet does not correspond with the maximum spouting area of the nozzle. As the jet area is increased, the power increases up to a certain point and then diminishes with further increase of area. To avoid such behaviour for Pelton wheels supplied through long pipe-lines, careful adjustment of the different dimensions, such as pipe diameter and normal ratio of nozzle area to pipe area, are necessary. As far as the writer is aware, much still remains to be accomplished in this direction to guide the designer, so that the clumsy expedient of fly-wheels may not be necessary to prevent hunting.

Speed Regulation of Prime Movers and Connected Machinery.

If the connection between a prime mover and the machinery which is driven by it be positive, any alteration

[1] *See* Article on "Governing of Pelton Water Wheels" by Professor Goodman, *Engineering*, Nov 4th, 1904.

in the speed of the one implies a corresponding change in the speed of the other, and as it is chiefly with such connections that the electrical and mechanical engineer has to deal, it is sufficient to assume that slip is non-existent, even in cases where the prime mover is connected to the driven machinery by belting or other connection, which might, if improperly designed for the power to be transmitted, have a certain amount of slip. Except with direct connected machinery the angular velocity of the driven unit is not necessarily equal to that of the driver, but the linear velocity of points upon driver and driven pulleys must be the same when connected by a belt. Consequently, for ordinary connected machinery, either the angular velocity of the prime mover must be the same as that of the driver; or, there may be equality between the linear velocities; or again, in special cases, both may be equal. Unless otherwise stated, angular velocity will be referred to in what follows, for the linear velocity of rotating machinery does not express clearly the speed at all, but only that of one point of the rotating mass, while angular velocity is the same for all parts of a wheel, shaft, or any mass rotating about an axis.

If any source of power, such as a water turbine, be applied to drive a machine, any constant speed at which the connected pair revolves will necessarily imply that the torque of the turbine is equalled by the opposing torque of the machine, together with that due to the frictional resistances. This speed may be termed the normal speed when the turbine or prime mover is working at full load, and any departure therefrom implies that the effort of the prime mover or the opposing resistances have changed. In

practice many conditions may arise to upset the balance between the torque of the prime mover and the driven machinery, and consequently the unit will accelerate or fall in speed until a new balance is maintained, and the function of a governing arrangement is to restore the balance as rapidly as possible when from any cause it is disturbed.

This disturbance may be caused by a drop or increase in the energy which the prime mover puts into the connecting shaft, or it may be due to a variation in the resistance offered by the connected machinery. In either case the result is that the unbalanced torque acts to accelerate or retard the rotating mass until an opposing torque is developed to counteract it, when acceleration ceases and the speed again becomes uniform, either greater or less than the original speed, as the case may be.

As an example, supposing a turbine be geared direct to an electric generator running at 300 revs. per minute, and that the torque or turning moment exerted by the turbine at full load be 800 lb.-ft. This turning moment, at constant speed, is partly resisted by the reaction of the strong magnetic field in the dynamo, and partly by friction losses in bearings, wind resistance, etc., and together they combine to form an opposing torque of 800 lb.-ft., equal to that of the turbine, and the speed is therefore uniform in consequence. The horse-power of the wheel at this speed would be—

$$\frac{300 \times 2\pi}{33{,}000} \times 800 = 45{\cdot}7.$$

Imagine that the gate of the turbine is closed down to such an extent as to reduce the torque of the turbine by

25 per cent. There will now be an unbalanced torque of 800–600 = 200 lb.-ft., which will oppose the motion of rotation, and which will reduce the speed until a new balance is effected between the driver and driven machinery. The time elapsing before the speed is again uniform will depend upon the law connecting the resistances of the driven machinery with the speed. As frictional resistances increase with the speed, an addition to the power of the turbine will result in an acceleration until these resistances have increased to such an extent as to effect a balance with the effort of the turbine. In the same manner a reduction of the turning moment will result in a declining speed, which will only be arrested when the balance between the two is again consummated. The function of a governing arrangement as applied to the prime mover is therefore to maintain at all times the balance between the turning effort of the turbine and the sum of the turning efforts opposing rotation when the unit is running at the definite speed for which it is designed. Complete success is only attained by an absolute invariability in the speed, but this is not to be expected from any governing arrangement, for it is only by an alteration in the speed that the governor is brought into action at all.

The device known as the pendulum governor, consisting of rotating weights which occupy various positions according to the speed, forms the basis of turbine speed regulation. As these weights occupy a definite position for each speed, it is clear that absolutely constant speed is impossible with any governor using the pendulum principle as the key to its operation. In some forms of high-speed engine the governor is brought into action by a variation in the load. In such engines the belt pulley, instead of being keyed on to the shaft,

is attached thereto through the medium of springs, and as the belt pull, acting tangentially to the pulley, varies, the angular position of the wheel upon the shaft also varies, and this movement is communicated to the valve gear and changes the cut-off of the valve so that the turning effort of the engine may be increased or diminished proportionally, and thus a balance maintained. This form of governor is also subject to the same limitations as the pendulum governor, for before it is brought into action a change

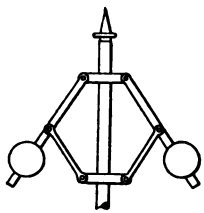


FIG. 51.—Unloaded governor.

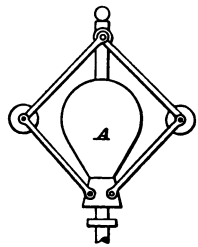


FIG. 52.—Loaded governor.

in the load or in the turning moment of the engine, and consequently in the speed, must have occurred. The pendulum governor, as used for hydraulic regulation, consists usually of two weights attached to a central spindle, and capable of being rotated, the position of the weights, which move in or out as the speed of rotation is diminished or increased, determining the position of the regulating mechanism. The governors may either be "unloaded" as shown in Fig. 51, or "loaded" (Fig. 52). In the latter case a mass A is attached to the linkage to which the two weights are attached, and this moves up and down upon

the spindle as the weights are moved out or in. This weight has a very important influence upon the action of the governor, which can best be understood by referring to the forces which act when the weights are rotating about the spindle.

To take a simple case. If a weight be suspended by a cord and rotated uniformly, so that the cord describes a cone, the position occupied by the weight, *i.e.*, the distance of its centre of gravity vertically below the fixed point of attachment of the cord, will be such that the forces acting upon the weight are balanced. These forces are the tension in the cord, the attraction of gravitation, and the centrifugal force. The position occupied by the weights is independent of their actual mass, and if h be the vertical distance of the centre of gravity of the weight below the point of suspension, corresponding to an angular velocity a, it will be connected to the angular velocity by the relation :—

$$h = \frac{g}{a^2} \text{ or } a = \sqrt{\frac{g}{h}}.$$

This shows that the height h varies inversely as the square of the angular velocity, or that the angular velocity varies inversely as the square root of the height of the weight of the governor.

In the actual governor the ideal string is replaced by a linkage and a collar upon the spindle to which the regulating rod is attached, all of which exert frictional resistances, which may be regarded as a load acting to oppose motion in either direction. The sensitiveness of a governor is shown by the movement which takes place for a given change in the angular velocity of the weights, and this varies for the position of the weights. When the weights

are far out, under the influence of a high speed, a change of a certain number of revolutions in the velocity exerts a comparatively small influence upon the height as compared with a change of the same number of revolutions when the speed is low. This is shown by the following table, in which

a	h.	Rise $(1-h)$.	Differences.
1·00	1·000	0	
			0·173
1·10	0·827	0·173	
			0·133
1·20	0·694	0·306	
			0·102
1·30	0·592	0·408	
			0·082
1·40	0·510	0·490	
			0·066
1·50	0·444	0·556	
			0·053
1·60	0·391	0·609	
			0·045
1·70	0·346	0·654	
			0·037
1·80	0·309	0·691	
			0·032
1·90	0·277	0·723	
			0·027
2·00	0·250	0·750	
			0·023
2·10	0·227	0·773	

a is increased in successive steps by the addition of 10 per cent. The second column shows the corresponding value of h, which decreases as a is increased. The third column $(1 - h)$ gives the vertical height that the weights have risen above the plane that they were at when $a = 1$. The differences are shown in the fourth column, and it will be seen that, as the speed is increased, the rise becomes less

for given increments of speed. The sensitiveness of a governor is therefore greater at the lower speeds, as the movement imparted to the valve is greater than at the higher speeds when an alteration of speed takes place.

The inevitable friction in the joints of the governor, and also at the collar, together with the varying angularity of the links, renders an exact mathematical expression for the height very complicated, and at best only approximations can be made to the true height for a certain speed. It is not necessary to enter into the theory here, but the influence of loading a governor should be referred to, as it has immediate practical applications. The sensitiveness may be greatly increased by the addition of a load, so that the motion of the loaded governor for a given change of speed is greater than in the case of the unloaded. The approximate increase in the sensitiveness by the addition of a weight $2W$, moving nearly at the same velocity as the ball, is :—

$$1 + \frac{W}{B} \text{ where } B \text{ is the weight of one ball.}$$

The sensitiveness is therefore increased in the ratio of $1 + \frac{W}{B} : 1$. Thus if $W = B$ the sensitiveness is doubled.

With an unloaded governor a speed is soon reached which gives too small a height, and therefore it is necessary to load the governor if it is required for high speeds. By this means a greater height for a given speed is attained, and a greater movement for changes in speed. Springs are also employed on pendulum governors instead of weights, and for fly-wheel governors they are used to balance the centrifugal force. The proper weight to be given to the balls

will depend upon the amount of variation from the mean speed that is allowed before the governor begins to act, for it must exert a force at the collar greater than the friction and resistances.

The pendulum governor is therefore the chief means by which an unbalanced condition between propelling effort and resistance is corrected in any motor attached to machinery. But there is also a valuable aid to the maintenance of a constant speed in the inertia of the rotating parts, or what is known as "fly-wheel effect," and this has a steadying influence, as the inertia of the parts prevents too rapid an acceleration when the balance is, for any reason, upset. The fly-wheel, or rotating mass, which includes in the case of a turbine plant the turbine itself and connected machinery, must be such that the speed shall not be too rapidly increased or diminished by any changes of load or turning moment imparted by the motor. On the other hand, it is wasteful of power and costly in maintenance to have the mass of the rotating parts more than is necessary, and the attainment of a correct proportion for these masses is a problem which is often shirked by adding more weight than is necessary, and which remains, to outward appearances, solved, in the same manner that a foundation, which is more than sufficient to sustain the load which is to be put upon it, shows the excess in extra cost of useless material, while it serves the function of sustaining the building as well as if it were correctly proportioned.

Some general remarks upon the subject of rotating masses will serve to illustrate the action of the fly-wheel as a regulator.

The mass of a body, or its inertia, is measured by the

force necessary to impart to the body a given acceleration, and the well-known equation, $F = M \times a$, applies universally. If, instead of the motion being in a straight line, the mass M is constrained to rotate about a radius r, a new relation exists which is represented by $T = I \times \omega$, where T, the turning moment, is equal to the product of the moment of inertia (I) and the angular acceleration ω. It is easily seen that if a mass be rotating about a fixed point the turning moment necessary to accelerate it at a given rate depends upon the radius at which the mass is rotating, and I, the moment of inertia, takes the place of M, the mass. This constant, which takes the same place in the dynamics of rotation as mass does in linear motion, is therefore the moment of inertia or $m\,r^2$, for a mass m rotating at a radius r. Engineers hardly ever speak of the moment of inertia; it is generally expressed as ton-ft.^{2}, lb.-ft.^{2}, or kilogramme-metres^2. The total moment of inertia of a wheel, or other rotating mass, is obtained by adding up the moments of the separate parts. This can be done for any geometrical figure, and the results are found in tables giving the formulæ for each figure. The engineer who is working problems in rotating bodies is chiefly concerned with rotating masses in the shape of cylinders or discs like grindstones or the rims of fly-wheels, and for these the moment of inertia, where r_1 is the outside radius of the disc, and M the mass, is

$$I = M \times \frac{r_1^2}{2}.$$

If the disc has a large hole in the middle of radius r_2, the moment of inertia of the rim is

$$I = M \times \frac{1}{2}(r_1^2 + r_2^2).$$

In this expression M is the mass of the rim.

The radius of gyration, which is sometimes referred to, is the distance from the axis of rotation at which, if the entire mass of the rotating body were concentrated, the moment of inertia would be the same. For a circle this distance is

$$\frac{r}{\sqrt{2}} = 0{\cdot}707r.$$

For simplicity it is usual to assume in calculations that the mass is concentrated at the mean radius or $\frac{r_1 + r_2}{2}$. For the same reason in calculations for weights of fly-wheels the spokes and hubs are neglected, as the rim, both by its position and relative weight, has an overwhelming effect, which makes the omission of the rest of little consequence.

As an example, a cast-iron fly-wheel of 6 ft. outside diameter has a rim 8 ins. deep and 6 ins. wide (parallel to the axis). Assuming 450 lbs. per cu. ft. as the weight for cast-iron $W = (28{\cdot}27 - 17{\cdot}10) \times 0{\cdot}5 \times 450 = 2{,}513$ lbs.

$$M = \frac{2{,}513}{32{\cdot}2} = 78{\cdot}1.$$

$$\frac{1}{2}(r_1^2 + r_2^2) = \frac{1}{2}(9 + 5{\cdot}45) = 7{\cdot}225.$$

$$I = 78{\cdot}1 \times 7{\cdot}225 = 564 \text{ lb.-ft.}^2$$

By making the assumption that the mass of the wheel is concentrated at the mean radius of the rim

$$I = 78{\cdot}1 \times \left(\frac{3 + 2{\cdot}333}{2}\right)^2 = 78{\cdot}1 \times 7{\cdot}11 = 555 \text{ lb. ft.}^2$$

This approximation therefore leads to a value of the moment of inertia, which is, in this case, within 2 per cent. of the actual. This is close enough for practical

purposes, though it is but little extra trouble to obtain it correctly.

The calculation of the total moment is complicated in turbine units by the inertia of the water passing through the wheel. This is especially the case in large turbines passing a large volume of water, and therefore they tend to run more steadily than impulse wheels which do not contain much water. Bodmer states that with some turbines (depending upon the vane angle) velocity of flow decreases as speed increases, while with others the contrary

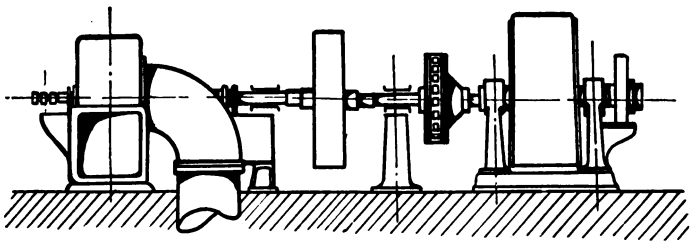


FIG. 53.—Turbine direct connected to generator, with fly-wheel.

is the case. The ratio of the radius of entry to exit also has an influence. For impulse wheels the velocity of flow is constant whatever the speed of the wheel may be. As most reaction turbines are designed, the velocity of flow increases as the speed increases. In direct connected units it often becomes necessary to increase the inertia by the addition of a fly-wheel, as shown in Fig. 53, which is sometimes placed between the turbine and generator, and sometimes upon an extension of the shaft outside the out board bearing. This arrangement necessitates a heavier shaft to resist the bending moment.

The armatures of electric generators are of necessity very heavy, and supply much of the needed inertia. In allowing for them they may be taken as a solid cylinder of metal of the air gap diameter, with an allowance for the ventilating ducts. With revolving field generators the calculation is not so simple, but the writer's practice is to omit the spokes and hub and to calculate the weight of coils and poles as if constructed of solid metal. This gives a fair approximation to the true value.

The coupling of an electric generator to a steam or gas engine is attended with special disorders which the hydraulic engineer does not experience. These are due to the uneven turning moment of the prime mover throughout the revolution, caused by the varying angularity of the crank, and the expansion of steam or gas in the cylinder giving rise to a varying pressure. The uniform turning moment of the hydraulic turbine is a distinct advantage, by enabling the rotating mass or fly-wheel effect to be kept lower than that required for a steam engine. For this reason turbines may be advantageously used to operate alternators in parallel with greater success than attends steam or gas-driven units for the same mass in the rotating parts. The requirements of successful operation of alternators in parallel are such that the angular deviation of the armature from the normal position must not be greater than a certain fraction of the pole-pitch. This allowance differs, but among the large electrical companies 1-200th is a usual figure.

The unit shown in Fig. 53 consists of a turbine driving an alternator direct, with the exciter on the end of the shaft. If a fraction of the load is suddenly thrown off, as by the opening of a switch in the circuit or distribution system,

the speed tends to rise, and supposing that there was no governor to restore the balance, the rate at which the speed would rise would be inversely proportional to the moment of inertia of the entire rotating mass. The addition of a fly-wheel will therefore serve to check the rate of change of speed, but cannot restore equilibrium. This latter function belongs to the governor. The difference between the fly-wheel and governor is, therefore, that the former allays the rate at which the speed changes when equilibrium is upset, and the governor effects the cure. The rate of change of speed (acceleration or retardation) may be made indefinitely small by the addition of metal, but there is an economical limit to this. Where turbines are connected to mills, the load is frequently changed slowly so that the acceleration is not constant, but in electric installations the throwing of a switch means that the turbine is instantly relieved of a fraction of the load and the resulting acceleration would be constant. In making calculations, constant acceleration or retardation is assumed, as it is impossible to obtain the law connecting the change in acceleration with change in the load owing to friction. The specifications to which governors are designed to conform are generally stated in allowable percentages of speed variation from the normal, for fractions of the load applied or thrown off. A good water-wheel governor ought to comply with the following conditions which would be considered necessary for the satisfactory operation of direct current generators, in conjunction with sufficient fly-wheel capacity to allow the governor time to act before the speed oversteps the limit.

With sudden variations of load of 25 per cent. the change in speed ought not to exceed 3 per cent.

With 50 per cent. load variation, speed variation 6 per cent.

With the entire load thrown off, speed variation 10 per cent.

Also with the load gradually thrown off during a period of not more than ten seconds the variation in speed should not exceed 4 per cent.

These specifications, if adhered to, enable a generator to be coupled to a turbine and used for arc and incandescent lighting without the necessity for storage batteries. With hydraulic governors they are frequently surpassed, and on a large installation in Switzerland better results have been obtained for variations of 25 per cent. and 50 per cent. in the load. In such a case unlimited power is at the disposal of the engineer, so that the gates are operated rapidly. With small installations this is not possible if the power has to be derived from the wheel, which cannot be drawn upon suddenly for the power necessary for regulating.

In order to comprehend the manner in which the speed changes with alteration in the load we must employ some suitable formulæ derived from elementary principles. If a shaft be rotating uniformly and transmitting power from the driver to driven machinery the relation between the torque, or twisting moment, in pound feet and the horse-power is expressed as

$$H.\text{-}P. = \frac{2\,\pi\,N}{60 \times 550} \times \text{torque},$$

from which $$T' \text{ (torque)} = \frac{5251 \times H.\text{-}P}{N}$$

N being the number of revolutions per minute.

If ω is the angular acceleration per second—*i.e.*, the gain

in revolutions per minute in one second—and p is the fraction of the load suddenly thrown off,

$$\omega = \frac{5251 \times H.\text{-}P.}{N \times I} \times \frac{60}{2\pi} \times p,$$

or

$$\omega = 50{,}140 \frac{H.\text{-}P.}{N \times I} \times p,$$

I being the moment of inertia of the rotating mass.

If the fraction of the load p be suddenly thrown off, as for instance when a switch is opened, the opposing unbalanced torque of the turbine, if allowed to persist, will cause the speed to increase as given by this relation. As an example, a direct connected unit as shown in Fig. 53 is running at 180 revolutions per minute, 75 h.-p. being transmitted from turbine to generator. With the generator suddenly relieved of 25 per cent. of the load, the increase in speed can be calculated when the moment of inertia is known. This is found to be 20,120 (mass units), therefore :—

$$\omega = \frac{75}{180 \times 20{,}120} \times 0{\cdot}25 \times 50{,}140 = 0{\cdot}26.$$

If no correction be made in the driving power by a governor, the speed will therefore have increased to $180 + 0{\cdot}26 \times 60 = 195{\cdot}6$ revolutions per minute in one minute, or about 8·7 per cent. With greater fractions of the load thrown off, the rate will be correspondingly increased. A certain correction is automatically made by the increase in the frictional resistances, especially wind resistance, which increases with the speed, and this tends to keep down the speed, but its effect may properly be neglected.

The increase in the speed, expressed in revolutions per minute, and percentage of the normal speed are plotted

against time in seconds on the curve (Fig. 54). For example, with 50 per cent. of the load suddenly thrown off, it is seen that the speed has increased 5 per cent. in 17 seconds, the increase being proportional to the time, and for 25 per cent. the increases are one-half those for 50 per cent.

If a disengagement governor were to be utilised to check the speed of the unit at 2 per cent. variation when 50 per cent. of the load was thrown off, we find from the curve that it would have to come into operation before 6·5 seconds had elapsed since the acceleration began. Such a governor would never be employed to control the speed, for it is necessary that a check should be put upon the acceleration before the velocity had increased to such an extent, and this the hydraulic and mechanical continuous governors do gradually from the moment the speed begins to rise above the normal. The action of a disengagement governor is shown by the dotted curve, and that of a continuous governor by the full line. Supposing 50 per cent. of the load be suddenly thrown off, the speed rises in accordance with the straight line, the slope of which shows the rate of rise. When the speed has increased 4 per cent. (to 187·2 revolutions per minute), which will be 14

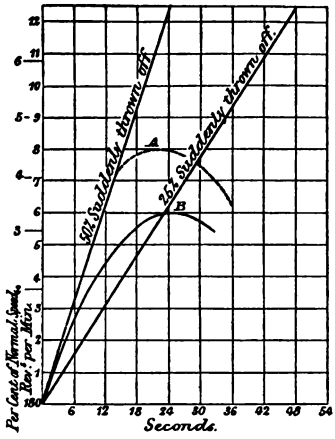


FIG. 54.—Acceleration curve.

seconds after the load is diminished, the governor acts, and the gates being gradually closed, the rate of gain in speed drops in consequence of the reduced driving torque. During this period the speed still rises, but at a decreasing rate, the point A upon the dotted line showing the maximum speed (188 revolutions per minute). As the driving torque has by this time been lowered by the shutting of the gates to such an extent that the resistance preponderates, the speed again drops as shown by the dotted curve. This drop is again checked when the governor goes out of action. The speed curve for a continuous governor is shown as a full line, the rate of change in the speed beginning at the moment the load is thrown off, the maximum speed (186 revolutions per minute) being attained in 24 seconds (point B on the curve).

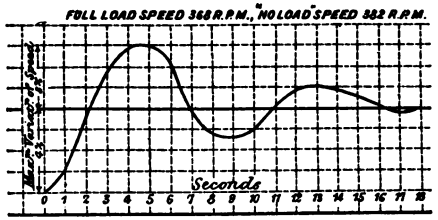


FIG. 55.—Action of continuous hydraulic governor.

Fig. 55 illustrates the action of a continuous hydraulic governor, and is a curve derived from a test made upon a 500 brake horse-power turbine at an electricity works in Switzerland, for which the author is indebted to Messrs. Escher, Wyss, & Co. The full-load speed of the turbine is 368 revolutions per minute, and the "no-load" speed 382 revolutions per minute. The curve shows the effect upon the speed of suddenly throwing off the entire load. The abscissæ are seconds, and the ordinates percentages of the speed, the distance between the horizontal dotted lines

being 1 per cent. The curve shows that the actual speed oscillates about the "no-load" speed like a damped vibration, each successive rise and fall being less than the preceding, until, about 18 seconds after the load is thrown off, the variation from the true "no-load" speed is very slight. This represents in general the action of the continuous governor upon the speed, which oscillates about a mean value.

A turbine by the same Swiss firm, working under a head of 14 metres (45·9 ft.) was tested for regulation with the following results. The wheel was a Francis turbine rotating upon a horizontal axis, and giving at maximum load about 120 h.-p. The tests were made by a brake, and for a variation from 82 to 34 brake horse-power the increase in speed was 3·64 per cent., *i.e.* from 330 to 342 revolutions per minute. For a variation of 10 h.-p. (72 to 62) the speed variation was 1·78 per cent. Expressed as percentages of full load, the variations show that the regulation was good. The governor operated upon the guide vanes of the wheel and was actuated by a hydraulic cylinder supplied with water at the pressure due to the head.

Types of Water Wheel Governors.

The earliest type of water wheel governor, which was applied to water wheels as well as to turbines, was a disengagement governor, which only came into action when an assigned departure from the normal speed was attained. Until this variation from the normal speed of the wheel or turbine was reached the pendulum governor was out of gear entirely with the mechanism for moving the gates, and it was only when a considerable displacement from the

mean position of the weights was reached that the governor, by automatically engaging a mechanism, effected the opening or closing of the gates. These governors are still used even for electric work, and with some success when the fly-wheel effect is carefully proportioned to the governor.

Figs. 56, 57, and 58 show types of disengagement governors. In that known as the Scholfield governor, there are the usual pendulum weights with spindle and collar. The shaft D (Fig. 56) is connected with the running machinery, so that the eccentric E, which is keyed to it, shall be constantly in motion. At the end of the eccentric rod is a pin projecting from a rocking lever which carries two pawls, P and R, at its extremity. These pawls are arranged to engage the spur wheels A and B, but are opposed to each other so that they tend to rotate the shaft C in opposite directions when one or the other is allowed to engage with the spur wheels. The shaft C is connected with the regulating gate, and a rotation in one direction shuts off the water, and in the other opens the gate. S is a shield which is moved to the right or left by the action of the governor, and which raises one or other of the pawls off the gears and therefore allows the shaft C to be rotated by a succession of movements according to the revolutions of the eccentric. When the speed is normal the shield is in

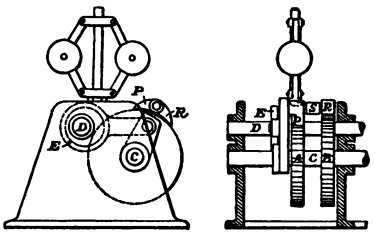


FIG. 56.—Disengagement governor.

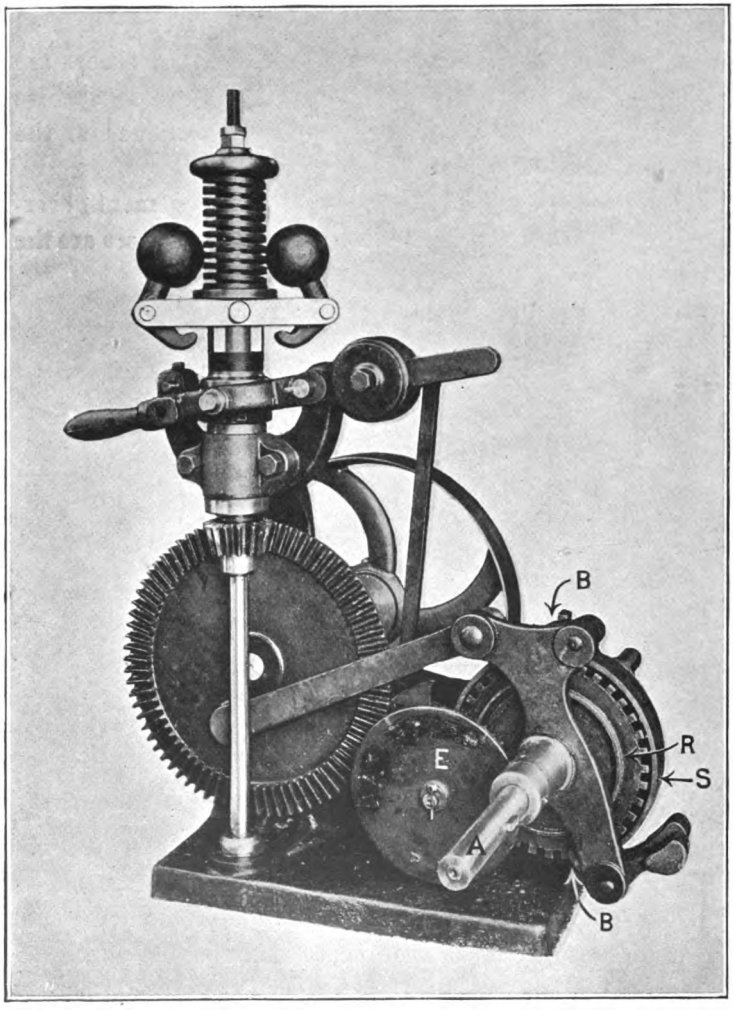


FIG. 57.—Disengagement governor.

mid position and has no effect, so that the shaft C vibrates back and forth without any appreciable change on the gate. The sensitiveness of the governor would be measured by the distance the shield S has to move before coming into action with a pawl.

Fig. 57 shows a governor of this type as made by Messrs. Gilbert Gilkes and Co., by whose courtesy I am enabled to produce the following description :—

Motion is imparted from the governor to the guide blade shaft A by means of two sets of pawls B B (one set for opening and one set for closing) which work on to a ratchet wheel R keyed on to the guide blade gear shaft A.

The position of the governor balls decide which set of pawls shall be in action. If the speed is high and the balls out, then the governor sleeve rising interposes a silencer or shield S between the opening pawls and the ratchet wheel. If the speed is low, the shield is interposed between the closing pawls and ratchet wheel. When the turbine, and consequently the governor, is at its normal speed, then the shield does not allow either set of pawls to work, and the guide blades are therefore stationary. It will be seen that, should the guide blades be full open, and the turbine for some cause below speed, the governor would endeavour to open the guide blades still further, and so break something. Such a contingency is provided for by a knock-off arrangement, which consists of a scroll cast on to the ratchet wheel. As the ratchet wheel is revolved the scroll causes the wheel E to be rotated.

Attached to E are two levers, so arranged that, when the guide blades are full open, one of the levers catches a pin and lifts the governor sleeve up, and of course at the same

time causes the shield S to be interposed between the two pawls and the ratchet wheel, thus preventing any further opening of the guide blades. When the guide blades are shut, the other lever acts in a similar manner, pulling down the governor sleeve, and so preventing the guide blades from closing any further.

Fig. 58 shows the Hartford governor, which is a continuous governor in the sense that any change in speed whatever is met by a corresponding action of the governor towards adjustment. A belt driven by the water wheel passes over two pulleys, one of which is a cone pulley. The belt is usually maintained tight by guide pulleys not shown. Each of these pulleys drives a bevel wheel (A and F) which mesh with the wheels D and B, and these wheels revolve in opposite directions. D and B are connected by sleeves with two similar bevel wheels which engage with C, and when revolving with the same speed in opposite directions C merely revolves upon its own axis. When, however, owing to the position of the belt upon the cone pulley, D and B have different speeds, the shaft to which C is keyed moves round and, being connected to the gate mechanism, regulates the supply of water. The position of the belt upon the cone pulley, and hence the differential motion of the bevel gears, is regulated by the belt shifter

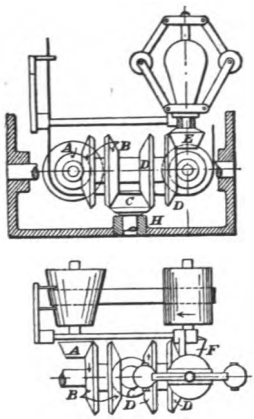


FIG. 58.—Hartford disengagement governor.

which is in connection with the pendulum governor. At the normal speed the belt is upon a diameter of the cone pulley equal to that of the cylindrical pulley, and hence there is no movement of the gate. The slipping of the belt, and the difficulty of always keeping it parallel to the normal to the pulley shafts, renders this form of governor sluggish in action. The principle of the relay is adhered to in all these governors, for the force of the pendulum governor is only utilised to operate mechanism, either occasionally, as in the disengagement governors, or continually, as in the belt-driven arrangement just described.

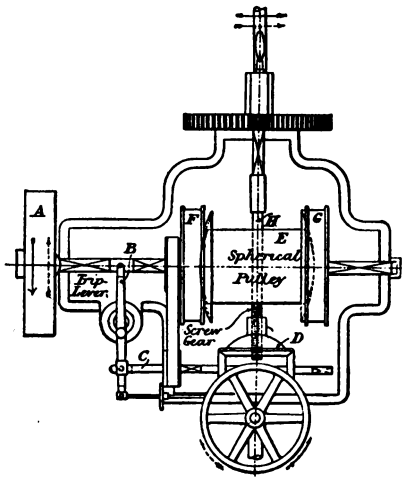


FIG. 59.—Lombard-Replogle mechanical governor.

Fig. 59 shows another form of mechanical governor, as made by the Lombard-Replogle Company, of Akron, Ohio. The pulley A receives motion from the turbine, and the governor balls are located within it, the position of which effects the motion of the trip lever B. This lever is poised upon a movable fulcrum, and the first act of the governor is to shift the fulcrum so as to allow the speed governor to rest at a slightly lower speed for a short time. The lever actuates the rod C, upon which are carried two

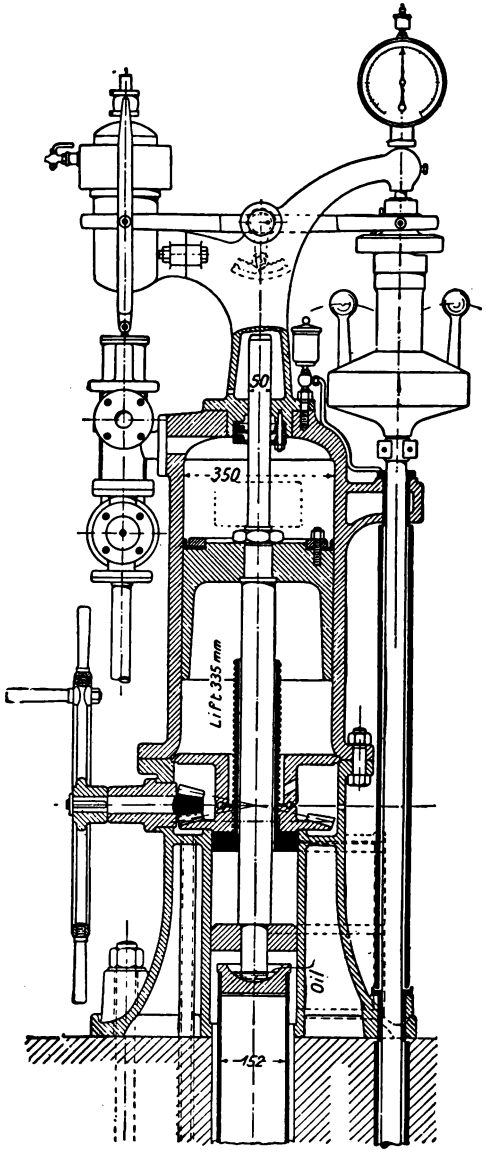


FIG. 60.—Niagara hydraulic governor.

bevel pinions which are arranged to engage with the nut gear D. The rotation of the nut D draws a spherical pulley E out of centre with the two concave discs F and G, and as these discs are rotated in opposite directions by a quarter-turn belt connecting the two pulleys, the spherical pulley is given a rotation in one or other direction, and at the same time it tends to centre itself by the rotation of the screw in the nut D. The rotation of the shaft H effects the closure or opening of the gates according to the direction of rotation. This differential motion is very effective, as it prevents the governor from over-running.

Hydraulic governors have almost entirely suppressed the mechanical governor for purposes of close regulation in electrical installations. The mechanical governor cannot be made as sensitive as the hydraulic, the valves of which are directly controlled by the pendulum. Under the name of hydraulic governors all forms of apparatus using the pressure of a fluid to operate the gates are included. The fluid is usually water, and is often under the pressure due to the head acting on the wheel, but sometimes, for installations working at low heads, the pressure is obtained by pumps, and also oil is substituted for water where a closed system of piping can be maintained. The hydraulic governor used at Niagara, which controls the speed of the 5,000 h.-p. turbines, is shown in Fig. 60. The position of the piston in the cylinder is regulated by the pressure of oil admitted through a valve controlled by the pendulum governor shown on the right of the illustration. Hand regulation is provided for by the bevel gears which are manipulated by the hand wheel outside the casing. This governor is placed on a platform about 125 ft. above the

turbine, but the heavy connections are balanced, and it is extremely sensitive. The regulation of these governors is such that with the entire load thrown off the generators as quickly as possible the increase in speed is less than 4 per cent. of the normal speed of 250 revolutions per minute. These governors are very much more satisfactory than the mechanical governors, and the speed variation under normal conditions (which would generally imply that less than 25 per cent. of the load would be at any time suddenly thrown off or on) is very satisfactory. The oil for operating these governors is supplied by pumps driven electrically.

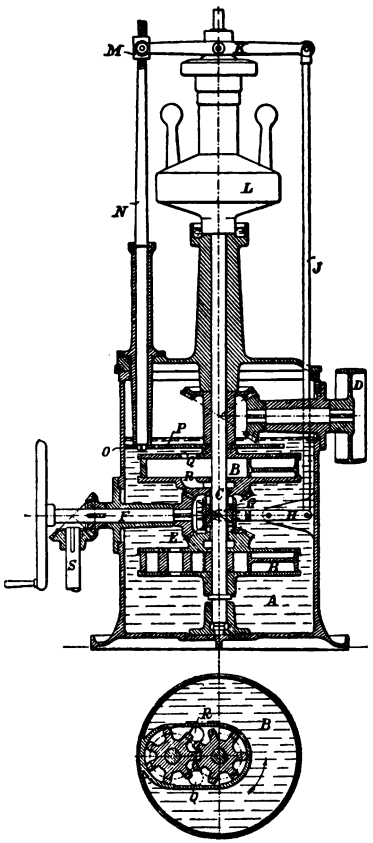


FIG. 61.—Hydraulic governor for low falls.

For small installations, operating under a low head of water, a governor of a different type is required, and the power consumed in operating the controlling mechanism must be derived from the turbine direct. If the wheel is of small power, that required to effect regulation makes a considerable

fraction of the whole, and must be considered in estimating the output of the plant. The governor shown in Fig. 61 is designed to control turbines working under heads too low for the utilisation of the hydrostatic pressure, and oil is used as the working fluid. There is an advantage in oil, as the valves and passages escape the chance of becoming choked with sand and impurities carried by water. If water is used, strainers of close mesh are required to intercept such impurities. The governor consists of a casing A, which is filled with oil, and two rotary oil pumps formed by close meshing spur gears in a tight case, one of each pair being keyed to the shaft C, which receives motion from the pulley D, driven direct from the turbine or counter shaft. These pumps are in mesh through a bevel gear with the bevel pinion E, which is keyed to a shaft communicating with the guide vane mechanism of the turbine. A valve G, worked by levers from the pendulum governor, controls the flow of oil to the pumps, and produces regulation in the following way:—When the pump rotates in the direction of the arrow, the suction is at Q and the delivery at R. If R be closed so that no oil can escape, the wheels cannot rotate, and the casing must rotate with the shaft C. The aperture Q communicates with the main body A, and R with the valve G, which is so arranged that a small motion of the governor closes the connection from the upper or lower pump aperture R, and thus locks either the upper or lower pump. The locking of a pump causes the shaft F to be driven in one or the other direction according as it is the upper or lower pump that is fastened, and this motion is communicated to the guide vanes. A very small movement of the pendulum causes action. The

oil in the chamber has to be renewed at intervals of some months, as otherwise it becomes thick, and is liable to clog the pumps and cause a slipping of the driving belt.

The governing arrangement for a Pelton wheel is shown in Fig. 62, as made by Messrs. Gilbert Gilkes & Co., Ltd., Kendal. The function of the governor is to move the needle

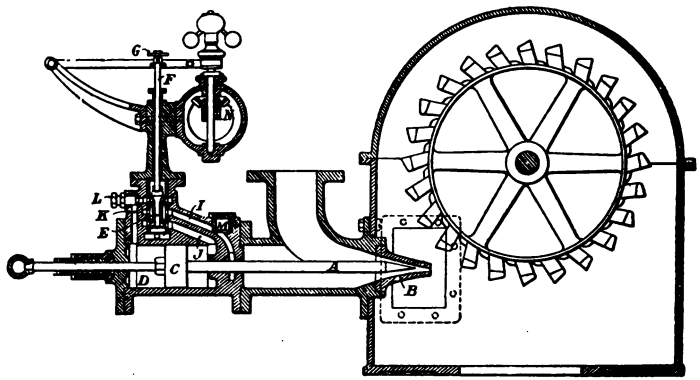


FIG. 62.—Pelton wheel governor.

in or out of the nozzle, and this is done by means of the pressure cylinder D, to which water is admitted through a double-ported valve E. The position of this valve is controlled by the pendulum governor. This method of governing is extremely delicate, as there is no heavy gate mechanism to put into motion, and the action of the regulating gate is immediate.

For mechanical governors of the best description, 10 to 15 seconds is required for the entire closing of the gates, but with hydraulic governors this time is reduced to 2 or 3 seconds. With the Pelton wheel governors

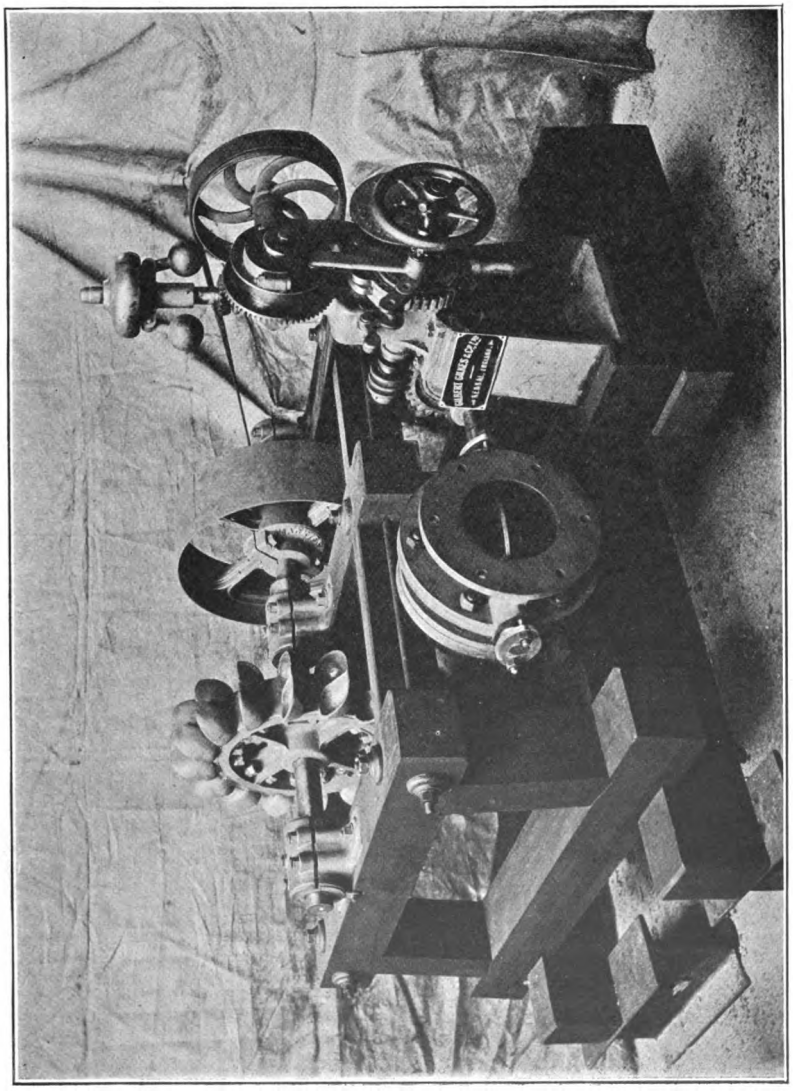
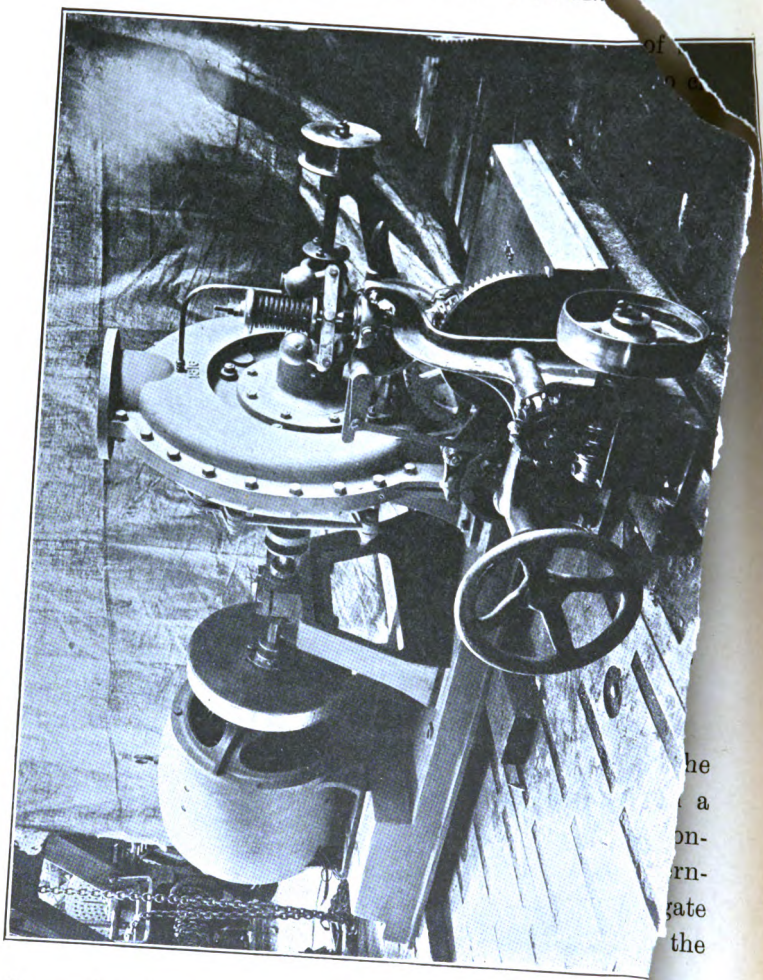


FIG. 63.—Pelton wheel controlled by mechanical governor.



the action is instantaneous. This rapid closing of the outlet to a long pipe line would involve danger; for this reason a deflection nozzle is preferable, in case the pressure within the cylinder is applied to

the nozzle. Sometimes a butterfly valve is employed in the supply pipe, which is directly actuated by the governors. Fig. 63 shows a mechanical governor by Messrs. Gilkes & Co. applied to the control of a small Pelton wheel through a butterfly valve. These valves as a rule cannot be used with long lines owing to the danger from water hammer, but when actuated slowly with a small mass of water behind them, they afford a simple means of controlling the flow to the wheel. The mechanism of this mechanical governor is simple, and consists of an eccentric which oscillates pawls which engage with toothed wheels in the manner already described (page 195). The eccentric is always in motion, being driven through the bevel gears from the pulley shaft. Fig. 64 shows the same arrangement applied to a small double vortex turbine. Steadiness of running is further ensured by the small fly-wheel between the turbine and dynamo.

Another form of regulating device for wheels of the Pelton type is the invention of Mr. Cassel. The wheel to which this device is applied consists essentially of two discs, upon the peripheries of which the buckets are situated. These discs are free to slide along the shaft. The tension of certain springs holding the discs together is balanced by a weight. When the wheel is running at normal speed the springs keep the discs together, so that the two halves of the buckets come together, and the impinging stream of water is caught on the dividing projections down the centre of the bucket. Should the speed of the wheel increase, the weights, flying outwards by centrifugal force, draw the discs asunder and allow part of the stream to pass through between the two halves of the buckets. The closing of the

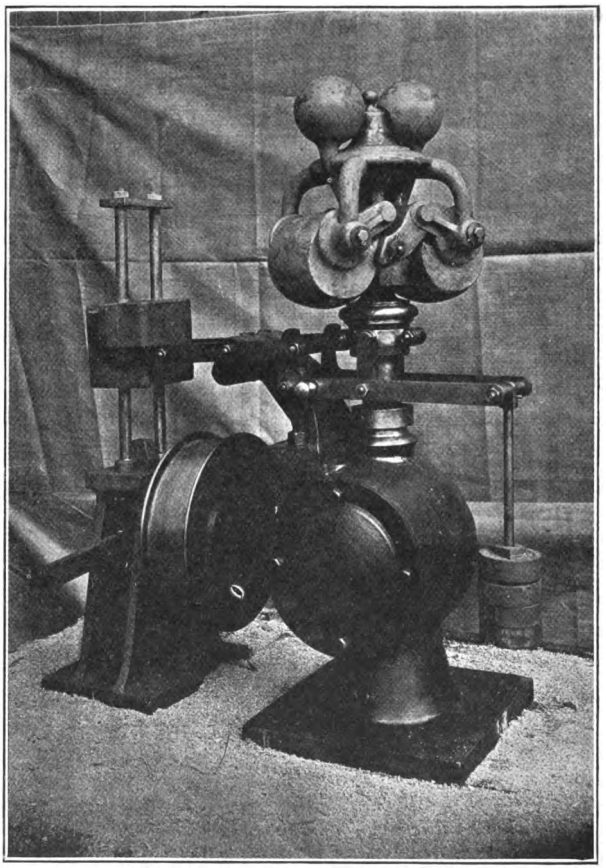


FIG. 65.—Emergency governor.

buckets takes place when the speed is again brought back to the normal, and so the wheel is controlled by allowing part of the stream to pass without doing useful work when the speed is above the normal. This controlling mechanism is therefore not unlike the deflecting nozzle, in that the

excess water is wasted when not required for driving the wheel.

Sometimes emergency governors have to be fitted in addition to main governors. In a plant for the Penhalonga Mines, in Southern Rhodesia, constructed by Messrs. Gilbert Gilkes & Co., emergency governors are employed, one of which is illustrated in Fig. 65. In this plant there are two Pelton wheels, which take their supply of water from a river 1440 ft. away, and 350 ft. higher than the power-house. The water is conveyed by a 22 in. riveted steel pipe, which branches into two 16 in. supply pipes inside the power-house, each branch supplying a 6 ft. Pelton wheel. The wheels are designed to run at 250 revolutions per minute, and each develops 375 h.-p. at normal speed. Each wheel is directly coupled to a three-phase generator supplying current at 440 volts and 50 periods when running at the normal speed. The pressure is raised by transformers in the power-house for the line. There are three governors fitted to these wheels. One is a sensitive hydraulic governor, which diverts the water from the wheels by means of steel slippers, which are machined to a knife edge, and the position of which in the jet is controlled by the main governor. The slippers are secured to levers keyed to a shaft passing under the covers of the wheels. One end of this shaft has a long lever keyed to it, which is worked from the governor. The governor has a spring loaded pendulum, which, through a system of levers, raises or lowers a valve controlling the water in the hydraulic cylinder, and thus moving the piston in or out, and the motion of the piston is communicated to the deflector shaft. A dash pot is provided, by means of which the speed of the

slippers may be regulated. This governor is driven by an independent motor, which is supplied with current from the switchboard. By placing both wheels under the control of one main governor, the alternators, which are in parallel, will tend to remain so. In the event of a short circuit upon one of the generators, the emergency governor comes into action and at once shuts off the water from the wheel.

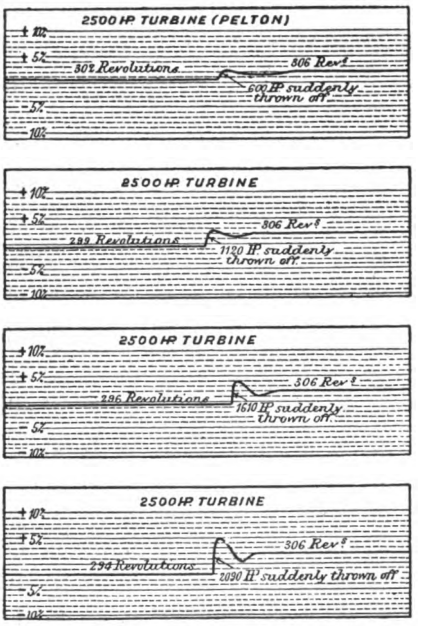


FIG. 66.—Test curves of 2,500 h.-p turbine.

The emergency governor consists of a loaded pendulum, which releases a cast-iron weight by means of a trigger if the speed should rise above a certain limit. The weight in falling operates the deflector and causes the full jet to pass the wheel, and until released by hand the wheel cannot be supplied with water.

From a test of an hydraulic governor made by Messrs. Theodor Bell and Co., Kriens, the curves shown in Fig. 66 are obtained. The turbine is a Pelton wheel of 2,500 h.-p., and the effect upon the speed of suddenly throwing off 600, 1,120, 1,610, and 2,090 h.-p. is shown by the four lines. The

distance between each horizontal dotted line represents 1 per cent. of the mean speed, and it will be seen by the last diagram that in throwing off about 83 per cent. of the load the maximum speed variation is about 7 per cent., and the vibrations of speed above and below the mean value are quickly damped out, so that the uniform speed of 306 revolutions per minute is soon attained. These curves are very instructive, for they show the excellence of the hydraulic governor as attained in modern practice.

CHAPTER IX.

WIND PRESSURE, VELOCITY, AND METHODS OF MEASURING.

THE currents of air which, in varying intensity and direction, blow over the surface of our globe have been employed by man from the earliest times as a motive power for propelling vessels over the seas, and, notwithstanding the introduction of steam, a large part of the carrying trade of the world to-day is done in ships propelled by the same natural element that filled the sails of craft thousands of years ago. It might therefore be supposed that the study of the winds and air currents which has been necessary to the mariner would have yielded results capable of scientific interpretation, and that this branch of meteorology would lead all the others in precise knowledge. But though the art of sailing has been practised for centuries, and our knowledge of prevailing winds over courses and at places on the surface of the earth is not without accuracy, certain features of the subject of air movements, which are chiefly useful to the power engineer, have defied the close attention which from time to time has been directed to them, and are still outside all scientific laws. It is only within recent years that studies of the wind have been made by experimental research, with a view to elucidating some of the points upon which the engineer especially desires knowledge, and about which we will treat in this chapter.

The wind, at a given place on the earth's surface, is

constantly varying in intensity and direction throughout the year. The change in direction may be either momentary, or it may be a decided change in the point of the compass from which it blows. The former changes are due to local influences, such as surrounding objects may exert by reflecting the wind from their surfaces, and the results of which are well known to the erecters of windmills, who have to choose a locality free from such disturbing influences. These sudden changes are seen in the oscillating movements of weathercocks and of the head of a windmill which is blanketed by buildings, trees, or other obstructions. The permanent changes in direction (permanent only in contradistinction to the above rapid changes), and which are accompanied by a corresponding change in the weathercock, have their causes in the varying distribution of heat on the earth's surface, which leads to upward currents of heated air at places, towards which the colder air moves to supply the vacancy thus created. While it is true that the contour of the surface of the earth at a locality has a great influence upon the prevailing direction of the wind at that place, at altitudes above the influence of the surface, the prevailing direction of the wind in many places is well defined, and at certain places these prevailing winds bear names such as the "trade" winds. In a mountainous country the wind naturally takes the direction of a valley either up or down, for the lateral ranges shut out winds which blow across the direction of the valley. The direction of the wind is different at different altitudes, the ocean of atmosphere being seemingly composed of layers or strata of air moving independently. Aeronauts take advantage of these currents of varying direction by rising

or descending until the balloon enters a current moving in the desired direction, though they are not always successful in finding one quite suitable. If their desires were always gratified, controllable air ships would have become common, and the North Pole would have been reached ere this.

But the surface of the earth presents such a rough and uneven bed over which a current of air has to pass that the current, instead of being made up of continuous parallel stream lines, becomes a boiling mass of eddies, vortices, and subsidiary currents, some of which even flow in the opposite direction to that of the main stream. It is out of this confusion that the physicist and engineer have been endeavouring to obtain some rules or laws which will guide them in predicting wind pressures from velocities and other important relations incidental to the flow of fluids. The motion of a leaf blown from a tree or other light object caught in the wind will illustrate the complex nature of the current, and how far from correct it is to assume that the movement of air is uniform and in parallel stream lines. Nor is there any evidence to show that the air current moves uniformly in the upper layers of the atmosphere, where, with the friction of the earth removed, such might be expected. There is, on the contrary, accumulated evidence to point to the conclusion that at any elevation the air moves in a manner quite different from that of water in deep rivers, which, though split up into eddies close to the bed or banks of the stream, flows along with a regular motion where such disturbing influence does not exist. The late Professor S. P. Langley, whose researches upon aerodynamics and the problem of flight form some of the most valuable contributions to the subject,

refers to the complex character of the air currents in one of his monographs.[1] While making some observations with a light anemometer, his attention was attracted by the peculiar behaviour of the instrument, which registered an extreme irregularity of velocity in a wind which would have otherwise been classed as a feeble breeze. These irregularities suggested to him the possibility that a so-called steady wind is in reality made up of variable and irregular movements of great frequency which the inertia of the usual form of anemometer would hide. He accordingly made an anemometer especially for the purpose of observing this point more closely, which weighed only 5 grammes and had the small moment of inertia of 300 gr.-cm.[2] By means of this instrument he was able to compare observations with those taken by an ordinary weather bureau anemometer, with the result that his former conclusions were confirmed. A comparison of the two anemometers showed that the wind was registered on the light anemometer as very variable, while the ordinary instrument did not show the small variations at all. The record was taken over a period during which the wind would travel two miles, and the following are some of the observed results.

In the words of the experimenter, "the velocity, which was at the beginning of the interval considered nearly 23 miles an hour, fell during the course of the first mile to a little over 20 miles an hour. This is the ordinary anemometric record of the wind at such elevations as this (47 metres) above the earth's surface, where it is free from the immediate vicinity of disturbing

[1] "The Internal Work of the Wind" (Smithsonian Contributions to Knowledge), by S. P. Langley.

irregularities, and where it is popularly supposed to move with occasional variation in direction, as the weathercock indeed indicates, but with such nearly uniform movement that its rate of advance is, during any such brief time as two or three minutes, under ordinary circumstances, approximately uniform. This, then, may be called the 'wind,' that is, the conventional 'wind' of treatises upon aerodynamics where its aspect as a practically continuous flow is alone considered. When, however, we turn to the record made with the specially light anemometer, at every second, of this same wind, we find an entirely different state of things. The wind starting with the velocity of 23 miles an hour, at 12 hours, 10 minutes, 18 seconds, rose within 10 seconds to a velocity of 33 miles an hour, and within 10 seconds more fell to its initial speed. It then rose within 30 seconds to a velocity of 36 miles an hour, and so on, with alternate risings and fallings, at one time actually stopping, and passing through 18 notable maxima and as many notable minima, the average interval from a maximum to a minimum being a little over 10 seconds, and the average change of velocity in this time being about 10 miles an hour."

These experiments were primarily conducted to throw light upon the problem of flight as exemplified by the soaring of heavy birds which, without apparent muscular effort, sustain their weight in a current of air. Without entering into a question which would be irrelevant to the subject, it may be said that Professor Langley concluded that the energy requisite for sustaining a mass in the air having a specific gravity many hundreds of times that of the medium was obtained from these pulsations in the

velocity. M. Mouillard, who has observed the actions of soaring birds very carefully, asserts that it is possible for them to advance against a wind without flapping their wings.[1] The solution of the problem of mechanical flight may, therefore, lie in the invention of a mechanical arrangement which will simulate the action of birds, by utilizing the rapid fluctuations in the speed of the air current of which a so-called steady wind is composed, and from which they derive energy. The lesson to be drawn from these experiments and others is that, in dealing with the action of wind upon sails or wings exposed to its influence, we have a problem of extreme complexity to solve, and which is but little understood. We must therefore pass on to the practical questions of the measurement of wind velocity, while bearing in mind the limitations in our knowledge as proved by these experiments and others of a similar kind.

The intensity of a wind is either referred to by an arbitrary name such as "breeze," or else it is designated by velocity in feet per second, miles per hour, or kilometres per hour. The table on p. 218 gives, as far as the indefinite nature of the ordinary designation for wind will allow, the corresponding velocity in these units.

It is customary among English engineers to use miles per hour, and in metric countries kilometres per hour, where wind is referred to in connection with windmills and the like. Some manufacturers in the United States use feet per second.

The direct measurement of the velocity of the wind forms one of the principal records of a meteorological observatory,

[1] L. P. Mouillard, "L'Empire de l'Air," Paris.

VELOCITY.			Ordinary Designation.
Miles per Hour.	Feet per Second.	Kilometres per Hour.	
1	1·47	1·61	Barely observable
2	2·93	3·22	Just perceptible
3	4·40	4·83	Light breeze
4	5·86	6·44	
5	7·3	8·05	Gentle wind
6	8·8	9·66	
8	11·7	12·9	
10	14·7	16·1	Fresh breeze
15	22·0	24·1	Brisk blow
20	29·3	32·2	Stiff breeze
25	36·7	40·3	Very brisk
30	44·0	48·3	High wind
35	51·3	56·4	
40	58·7	64·4	Very high wind
45	66·0	72·5	Gale
50	73·3	80·5	Storm
60	88·0	96·6	Great storm
80	117·4	128·8	Hurricane
90	132·0	144·9	Tornado
100	146·7	161	

and self-recording instruments are employed at most stations from which a continuous record of the velocity (and direction)

of the wind is recorded for reference. These records are chiefly of value for predicting the weather over large continents, when taken in connection with barometric observations, and also for recording storms of unusual violence. The instruments with which observatories are usually equipped are of two kinds: (1) the Robinson anemometer (the invention of the late Dr. Robinson); (2) pressure recorders. Of these two instruments the anemometer is the one most used. It consists of four semi-spherical hollow cups at the ends of arms pivoted to a vertical axis and capable of rotation about the axis with slight friction. The wind, acting on the concave side of one cup, and the convex side of the cup at the end of the arm opposite, produces an unbalanced turning moment, owing to the difference in the pressure exerted on the convex and concave sides. The linear speed of the cups gives a measure of the velocity of the wind, and in the recording forms of the instrument the motion of the cups is imparted to a drum, through suitable gearing, the rotation of which is indicated by a pencil pressing against the sheet of paper by which it is covered. The pencil is simultaneously moved by clock-work longitudinally (parallel with the axis of the drum) so that the slope of the line shows the velocity of the wind, and the sheets are periodically removed from the drum and may be filed for reference. There are other forms of this anemometer less elaborate than this which integrate the revolutions, the figures being read off a dial directly. The standard anemometers of the Robinson pattern as used by the weather bureaus of different nations differ somewhat. The Kew pattern, which is the standard in Great Britain and in some other European countries, has

brass cups made of No. 21 gauge metal 9 ins. in diameter, and clamped to $\frac{3}{8}$ in. round steel arms with their centres 24 ins. from the axis of rotation. The weight of the cups is usually borne by steel wires attached to the top of the spindle which is extended above the plane of the cups. The United States Weather Bureau has adopted two instruments, one having brass cups and the other aluminium cups. They are 4 ins. in diameter and are mounted on square steel arms. The diagonals of the arms are set vertically and horizontally, and the centres of the cups are 6·72 ins. from the axis. The weight of the aluminium cups, including the spindle supporting them, is about 280 grams., and that of the brass cups 529 grams. There is another form of anemometer which was designed by Mr. Dines called the helicoid or air meter. It consists of a fan made of two aluminium blades like a screw propeller. One of these instruments, in which the fans have a radius of 3 ins., makes approximately two revolutions for each metre of wind registered by it. The weight of blades and spindle is about 15·5 grams. The necessity for low friction in the bearings has called forth the skill of the mechanician in making anemometers. The bearings of the best instruments are ball bearings or agate cups in which the weight of the instrument is supported, with means to keep them always well lubricated, and of such a pattern that they will run for a long time without attention or inspection, as anemometers are not always readily accessible, and consequently may be left for months without any attention.

The speed of the cups of the Robinson anemometer depends, of course, upon the velocity of the wind which is propelling them, and also upon certain constants depending

upon the instrument itself. A certain pressure is necessary to overcome the statical friction of the moving parts of an anemometer in its bearings, which means that, before an anemometer will begin to move at all, a certain pressure or wind velocity must have arisen. In an anemometer of rough construction, with comparatively great friction in the journals, this velocity may be considerable before the pressure is sufficient to cause acceleration of the cups; but with nicely made instruments the minimum wind velocity that is needed is very small. As the velocity of the wind increases the speed of the cups likewise augments, and for practical purposes the relation between the two may be taken as directly proportional, though in refined experiments, as we shall see, this is not quite correct. That is to say, if V be the velocity of the wind, and v the lineal speed of a cup, $V = a\,v + b$, where b is the constant before referred to, being the velocity of the wind which is just sufficient to communicate motion to the cups (if $v = o$ in the equation, then $V = b$). The constant a, which is the number that the speed of the cup must be multiplied by to obtain the wind velocity, was stated by Dr. Robinson to be three, or in other words, the speed of the cup is always one-third that of the wind. This factor has been used until recently by the Meteorological Office for reducing the anemometer observations to wind velocities, but it is now replaced by the factor 2·2, so that the cups of the standard Kew anemometer move with a speed somewhat less than one-half the velocity of the wind. This difference in the anemometer constant is accounted for by improvements in construction over the earlier instruments which have resulted in reduced friction, and also by the results of

accurate experiments. Mr. A. Lawrence Rotch, B.Sc., the accomplished director of the Blue Hill Meteorological Observatory, near Boston, Mass.,[1] has conducted some interesting anemometer comparisons and tests, the results of which are recorded in the "Annals of the Astronomical Observatory of Harvard College." As these experiments were carried out with great care, the results may be taken as authoritative in establishing some points concerning anemometers in general which are brought together for comparison in a convenient form, and from which some of the principal conclusions may be usefully extracted.

The object of these experiments was to determine the mean differences between anemometers used as standards in different countries, so that wind velocities in one country might be directly compared with those in another. Besides the types of Robinson anemometer used as standards in Great Britain and by the United States Weather Bureau, Dines helicoid anemometers were also tested. The mean differences that were determined are the differences between the movements recorded by the anemometers during an interval of six minutes (one-tenth of an hour). The instruments were placed 26 ft. above the roof of the building upon a mast with cross arms, and were subjected to the same wind influences, so that the readings could be directly compared and the relative sensitiveness obtained. Preliminary comparisons of several anemometers, of exactly the same weight and size, showed differences that could not be accounted for, and careful measurements did not reveal

[1] The kite and *ballon-sondes* experiments made at the Blue Hill Observatory upon air currents in the higher strata of the atmosphere are described in "Sounding the Ocean of Air" by A. L. Rotch.

sufficient variation in constructional detail between them to account for the differences in the observations. When, however, the round arms of the Kew instrument were replaced by flat arms with knife edges, the rate was found to have increased by 2 per cent. These tests proved the necessity of adhering to an absolute standard in the construction of an anemometer, even to the minutest details, if uniform accuracy is to be attained, or if a constant factor of reduction is to be universally applicable. The method of transmitting the motion of the cups to the recorder also affects the rate of the instrument. The most delicate arrangement is by the use of a mercury contact which closes a circuit periodically. Accurate experiments with whirling tables have been made by Mr. Dines and also by Professor Marvin. By means of a large machine 29 ft. in length, driven by a steam engine, Mr. Dines found that the factor for the Kew standard instrument was between 2·00 and 2·27, instead of 3, and that it was practically constant. The whirling table upon which such experiments are made consists essentially of an arm pivoted at one end and capable of being rotated. At the other, or free, end the anemometer is fastened, and the arm is rotated at a speed which can be measured exactly. The anemometer is therefore moving in a wind the velocity of which is known, and the effect on the instrument is regarded as the same, whether the anemometer be stationary in a wind of given velocity, or whether the instrument as a whole be moved through still air at the same speed. Mr. Dines concluded from his experiments that, if 2·10 were taken as the factor, it would be within 5 per cent. of the correct value, and possibly within 2 per cent.

The machine used by Professor Marvin was 28 to 35 ft. long and was turned by hand. It was employed for determining the constants of the United States Weather Bureau instruments. The following formulæ, deduced from the experiments, apply only to the Standard Weather Bureau instruments :—

$$(a)\ V = \cdot225 + 3\cdot143\ v - \cdot0362\ v^2$$
$$(b)\ V = \cdot263 + 2\cdot953\ v - \cdot0407\ v^2$$
$$(c)\qquad V = \cdot466 + 2\cdot525\ v$$
$$(d)\ \log V = \cdot509 + \cdot9012 \log v$$

in which V is velocity of wind in miles per hour, and v is velocity of the centres of the cups in miles per hour.

Formula (*a*) was computed directly from the whirling machine experiments; (*b*) is the same formula adjusted to open-air conditions (for the experiments were carried on in a large room) ; (*c*) and (*d*) are empirical.

Professor Marvin states that (*b*) is the formula which most closely fits the experiments for velocities of 0 to 40 miles per hour, as 32 miles per hour was the limit to which the experiments were carried. It will be observed that a term involving the square of the velocity is included, but this makes but little difference on the result, except in high and unusual winds, and the coefficient of v or 2·953 represents the constant for the instrument. Thus if $v = 7$, the velocity of the wind in miles per hour would be :—

$V = 0\cdot263 + 7 \times 2\cdot953 - 1\cdot994 = 18\cdot94$ miles per hour.

Formula (*c*) gives $V = 0\cdot466 \times 2\cdot525 \times 7 = 18\cdot14$ miles per hour.

The average velocities, as recorded by the Kew instruments, are about 13 to 18 per cent. higher than those of the Weather Bureau.

As a so-called steady wind is made up of alternate gusts and periods of calm, which is shown on a windy day by the rattling of window sashes; an anemometer only registers a mean value for a velocity which may be rapidly changing between extremes. The degree in which this variable wind velocity is recorded by the instrument is a measure of its sensitiveness, and experiments reveal that the lighter the instrument is, the more readily will it respond to rapid fluctuations of velocity, as might be expected from the comparative inertia of light and heavy cups. In this respect the U. S. Weather Bureau instrument, being of lighter make, is the more sensitive of the two, while the fan anemometers are still more sensitive. These rapid fluctuations of the wind, though of great scientific interest, in that many problems in aeronautics and mechanical flight may be directly referred to them, are not recorded by the anemometer, and as it is the pressure of the wind upon surfaces which chiefly concerns the engineer, the study of these

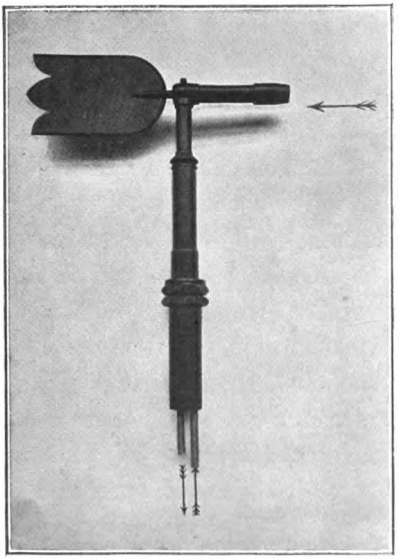


FIG. 67.—Head of Dines Wind Pressure Recorder (R. W. Munro).

complex phenomena is somewhat outside our present purpose.

Pressure-recording instruments indicate directly the pressure of the wind, which may afterwards be converted into velocity if necessary. One instrument of this type, which was invented by Mr. W. H. Dines, registers directly the pressure of the wind, so that the relation between velocity and pressure, about which there is so much ambiguity, does not enter into the record. The head, or part upon which the wind acts, is shown in Fig. 67. It consists of a short length of horizontal pipe, the open end of which is constantly kept turned against the wind by the vane. Two flexible metallic tubes connect the head with the recording apparatus, which may be placed at a convenient distance. The vane is pivoted upon the top of a vertical tube. This vertical tube is surrounded by another, the exterior of which is perforated by four rings of holes placed close together round the circumference. A float made of a specially-shaped copper vessel closed at one end is placed with the open end downwards in a closed vessel partially filled with water. The inside of the float is connected to the vane pipe through one of the flexible tubes, so that an increase of wind pressure causes it to rise in the water. The outer perforated pipe below the vane is connected by the second flexible tube with the closed vessel. The effect of the wind blowing across the open holes is to create a suction which reduces the barometric pressure within the vessel and thus contributes to raise the float in the water. The pen, or recording apparatus, is actuated by the float through suitable mechanism, and thus the air pressure is directly indicated upon a chart, one of

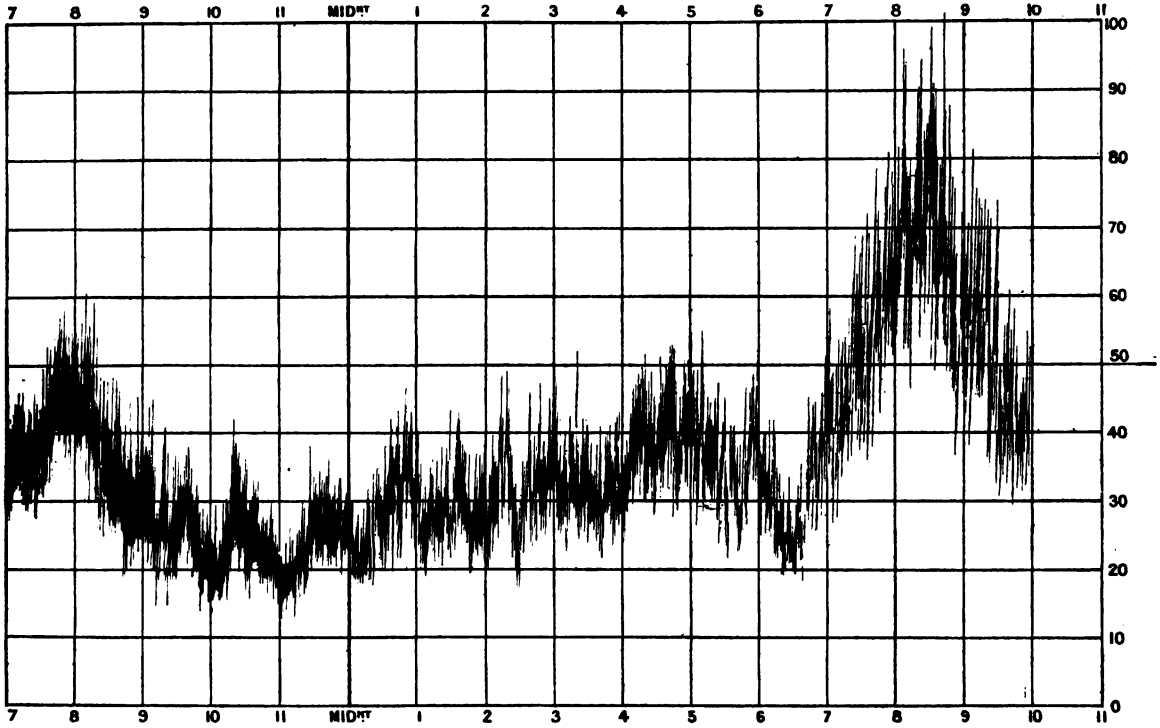


FIG. 68.—Wind Pressure as recorded by the Dines Pressure Gauge.

which is shown in Fig. 68, the paper being moved beneath the pen by clockwork. It will be seen from this record, taken in boisterous weather, that the pressure is incessantly changing. This is partly due to the inertia of the float and connections, and partly to the actual state of the barometric pressure within the tube caused by the rapid wave-like fluctuations in the actual barometric pressure at the mouth of the tube. When this instrument is used in cold climates the water is replaced by a mixture of spirit and glycerine, and the makers recommend the proportion of the two liquids to be one litre of glycerine to 1·28 litres of pure spirit. Pressure instruments will not always register the same for the same velocity of wind. With a low barometer and high temperature, which means that the air is less dense, the registration of pressure for a given velocity becomes correspondingly low. The maker of this instrument, Mr. Robert W. Munro, to whom the author is indebted for the illustrations of the apparatus, states that the corrections necessary from these causes are small.

The principal advantages of this instrument are its simplicity, and the fact that it cannot be injured by high winds; moreover, it is more sensitive than any of the rotation anemometers, and extreme wind velocities registered by it are more nearly correct than with rotation anemometers. Mr. S. P. Fergusson, who conducted experiments at the Blue Hill Observatory, states that at low velocities, below eight miles an hour, the mean velocity appears to be very nearly correct, but the instrument failed to respond to sudden changes of velocity, and there were often differences of 100 per cent. between

it and the cinemograph. In the lightest winds (below 5 miles per hour) it was less sensitive than any of the other anemometers except the pressure plate. Another defect of this instrument is its liability to become choked with snow in cold climates. Notwithstanding these objections, this experimenter says that it is an excellent instrument for indicating maximum velocities, and the results are more likely to have a constant value than those of any other anemometer.

An examination of anemometer records, as usually taken in an observatory, will reveal the prevailing direction of the wind, and the extreme variation in intensity and average velocity. The windmill engineer is not concerned to any extent with the direction of the wind, for all mills automatically adjust themselves so as to take advantage of wind coming from any point of the compass. It is the extreme wind velocities, and the average velocity over a period, with which he is chiefly concerned, also the way in which the wind varies from day to day or hour to hour. Before the erection of a windmill in any locality, a useful table may be made out from which the probable velocity of the wind, or the number of hours throughout the year that the velocity exceeds an assigned minimum, may be prophesied, as based upon the records of the nearest meteorological station. In hilly districts these prophecies are less likely to be accurate than over a flat country upon which an observing station is situated.

To illustrate the application of meteorological records to the purposes of the engineer, a record made at an observatory on the south coast of England during the month of January supplies the particulars as shown in the adjoined

table: col. 1 gives the day of the month; 2, the number of hours from midnight to noon that the velocity of the wind was 10 miles an hour or more; 3, ditto, from noon

Day of Month.	Number of hours midnight to noon, 10 m.p.h. or more.	Number of hours noon to midnight, 10 m.p.h. or more.	Total in 24 hours.
1	3	10	13
2	0	0	0
3	0	2	2
4	4	12	16
5	12	12	24
6	12	12	24
7	10	2	12
8	7	12	19
9	12	12	24
10	2	0	2
11	11	12	23
12	2	2	4
13	5	12	17
14	12	12	24
15	12	12	24
16	6	9	15
17	6	9	15
18	2	0	2
19	0	8	8
20	12	12	24
21	0	3	3
22	0	0	0
23	0	0	0
24	0	0	0
25	0	1	1
26	0	0	0
27	0	0	0
28	0	1	1
29	0	0	0
30	0	6	6
31	3	1	4

to midnight; 4, total number of hours in 24 hours that the wind velocity exceeded 10 miles per hour. Thus, from the point of view of the miller, the weather would

have been good, with only two bad days, from the 4th to the 17th, while the latter part of the month would be comparatively calm. While 10 miles an hour is arbitrarily chosen, it is not to be inferred that wind velocities above it are necessarily good, and those below too low for the working of windmills; but the number of hours of a wind of such velocity that can be counted on throughout the year is a fit measure of the suitability of the locality for the erection of a windmill. The danger of destruction by storms is not as great as formerly, since the automatically regulated steel mill came into use. The chief obstacle to the extension of windmills to all forms of industry is the unreliable character of the wind and the number of hours of calm that prevail at a time in even the most exposed situations. Mr. James Rickman, A.M.Inst.C.E., a well-known authority on the subject of the use of the wind in England as applied to the modern steel mill, informed the writer that the longest period of absolute calm in his experience was only three days. Of course, there are many days in which, though not absolutely calm, the wind does not attain sufficient velocity to work a mill, but even then the number of such consecutive days in the calmest season of the year is small. Mean velocities of the wind are not as instructive, from the engineer's point of view, as the number of hours that can be counted on for a working wind. At some places, particularly the West Indies, a high average wind velocity is the rule, but this is the result of alternate hurricane and calm—a most unsatisfactory condition from the engineer's point of view. The following table which is taken from the Meteorological Office reports shows the mean wind velocities at four stations

for every month in the year, as recorded by Robinson anemometers:—

MEAN WIND VELOCITY AT FOUR STATIONS FOR THE YEARS 1881-1905.

	Valentia.	Aberdeen.	Falmouth.	Kew.
January .	14·7	10·4	11·8	8·1
February .	13·8	10·3	11·6	8·4
March . .	13·1	11·5	11·5	8·8
April . .	12·0	10·6	10·6	8·4
May . .	11·1	9·4	9·4	7·6
June . .	10·2	8·7	8·7	7·0
July . .	10·1	8·9	8·9	6·5
August. .	10·7	9·0	9·0	6·7
September	11·1	8·7	8·7	6·2
October .	12·1	9·8	9·8	6·5
November	13·3	10·7	10·7	7·5
December .	14·6	11·8	11·8	7·9

Valentia, on the south-west coast of Ireland, which is the most exposed of the four stations, has a mean velocity for these years of somewhat more than 10 miles per hour, while the Kew station is considerably less than that figure. The high Atlantic westerly winds which blow over the west coast make this situation one of the best for windmills, while an inland station does not benefit to the same extent, as may be seen by comparing the other three with Valentia.

RELATION BETWEEN VELOCITY AND PRESSURE.

The engineer is chiefly concerned with the pressure that the wind exerts upon surfaces exposed to it, whether he desires to build structures to withstand the wind, or whether he is engaged in problems pertaining to windmills. By the

anemometer he may ascertain the velocity of a wind which, striking an exposed surface, results in a pressure upon it. What he most desires to know is the law that connects velocity and pressure, so that given the one he may readily deduce the other from it. The pressure which the wind exerts upon bridges, roofs, and all other forms of structure exposed to it necessitates extra precautions in construction, and it is important to know what this pressure may be so that the resisting powers of the structure may be sufficient to avert destruction in a high wind. The instruments for the direct measurement of pressure are not always available, and even if their readings could be relied on for accuracy, it is more convenient to be able to readily convert the common terms expressing the strength of a wind in velocity, direct into pressure. It is clear that the pressure exerted by a wind increases with the velocity, but it is not so evident that the pressure is not proportional to the velocity. Indeed, for a long time it was so regarded, until experiments conducted of late years have proved the contrary. If it were proportional to the velocity we should find that $P = a\,V$ where a is a constant which is multiplied by the velocity V to obtain the pressure P. Thus if a wind of 25 miles an hour resulted in a pressure of 3·5 lbs. per square foot, a wind of 50 miles an hour should produce a pressure of 7 lbs. per square foot, and the constant a would be 3·5/25, and $P = 0{\cdot}14\ V$. Such a relation is, in the light of recent experiments, far from correct, as it is found that the pressure increases more rapidly than the first power of the velocity, and while the exact function is not yet known, a nearer approach to accuracy than is given by a linear function has been attained.

The method of obtaining a value for the pressure of the wind upon an exposed surface for varying velocities necessitates (1) a correct measurement of the velocity; (2) a measurement of the pressure by direct means. By observing the relation between these two quantities for a sufficient number of values, it is possible to obtain the law connecting the two, so that by interpolation the one may be derived from the other for all values. The anemometer may be used for obtaining the first of these quantities, and pressure boards for the other. The pressure boards which have been used are simply plane surfaces placed normal to the direction of the wind, and the pressure is registered on a gauge, the board being connected thereto so as to measure the pressure directly. The size of the board has an importance that in the early experiments was not fully considered, for it affects the pressure per unit of area to a great extent. As the exposed surfaces of structures are generally larger than could be conveniently experimented upon, it would appear that the results of experiments with large boards would, by simulating actual practice more closely, have the most value.

The earliest formula connecting pressure and velocity is wrongly attributed to Smeaton, who obtained it from his friend Mr. Rouse, but he was so convinced of the accuracy of the relation that he adopted it, and it has been used in many countries since. If V be the velocity of the wind in miles per hour and P the pressure in pounds per square foot

$$P = 0{\cdot}005\ V^2$$

The following table gives in col. (3) the pressures worked out for various wind velocities according to this relation

and the curve (Fig. 69) is plotted with pressures as abscissæ and velocities as ordinates:—

$P = 0.003V^2$

(1) Miles per Hour.	(2) Feet per Second.	(3) Pressure per Sq. Ft. in lbs. (Smeaton).	(4) Pressure per Sq. Ft. Correct.
1	1·47	0·005	0·003
3	4·4	0·045	0·027
4	5·9	0·080	0·048
5	7·4	0·125	0·075
6	8·8	0·18	0·108
7	10·3	0·245	0·147
8	11·8	0·32	0·192
10	14·7	0·5	0·3
12	17·6	0·72	0·43
14	20·6	0·98	0·59
16	23·5	1·28	0·77
18	26·5	1·62	0·97
20	29·4	2·0	1·2
25	36·8	3·12	1·87
30	44·1	4·5	2·7
35	51·5	6·1	3·66
40	58·8	8·0	4·8
45	66·2	10·1	6·1
50	73·5	12·5	7·5
60	88·2	18·0	10·8
70	103·0	24·5	14·7
80	118·0	32·0	19·2
100	147·0	50·0	30·0

The accuracy of this formula has been assailed with success, and the latest experiments go to show that the pressures so calculated are somewhat too high for the corresponding velocities, and other formulæ are now regarded as more nearly true. The fact that the wind pressure varies as the square of the velocity is sufficiently proved, but the coefficient in the Smeaton formula should be smaller, as the best experiments clearly indicate.

The late Sir Benjamin Baker conducted a series of experiments at the Firth of Forth to guide him in designing

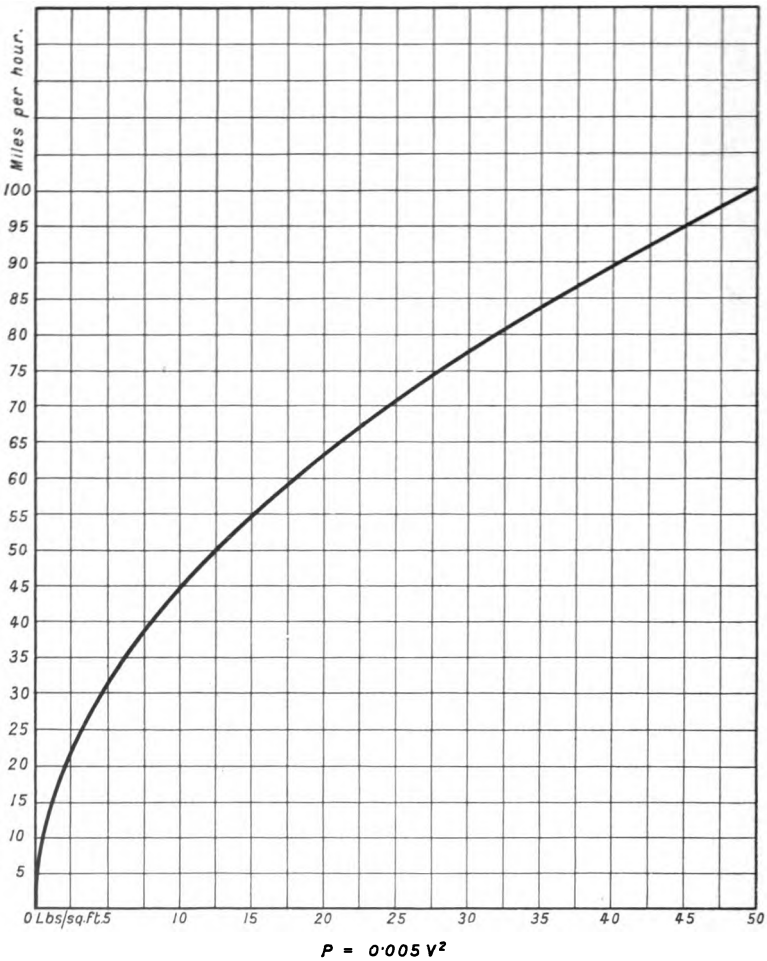


FIG. 69.--Relation between wind pressure and velocity (Smeaton).

the great structure that spans that estuary. He used three pressure boards, the largest of 300 sq. ft. and two smaller

ones 1·5 sq. ft. each. His object in using boards of such varied size was to ascertain the difference in unit pressures derived from small experimental boards, and large surfaces comparable with those that the bridge would present to the wind. After two years of experiments conducted with great care he arrived at the conclusion that if the wind bracing were proportioned to withstand a maximum pressure of 56 lbs. per square foot, the structure would have an adequate factor of safety, as such pressures would not be attained under the highest winds. According to Smeaton's formula such a pressure would correspond to a wind of 106 miles per hour, and though pressures of 70 lbs. per sq. ft. have been recorded in England they would be caused by momentary impulses which would have no effect upon a large and heavy structure. A tornado in the tropics at a velocity of 100 miles an hour might produce a steady pressure approaching 50 lbs. per square foot, but the practice of engineers in England and the United States is to design wind bracing for a pressure of 30 lbs. per square foot.

The experiments carried out by Mr. Dines with a whirling table, show that the coefficient in Smeaton's formula is too high and that the correct relation would be—

$$P = 0{\cdot}0029\ V^2$$

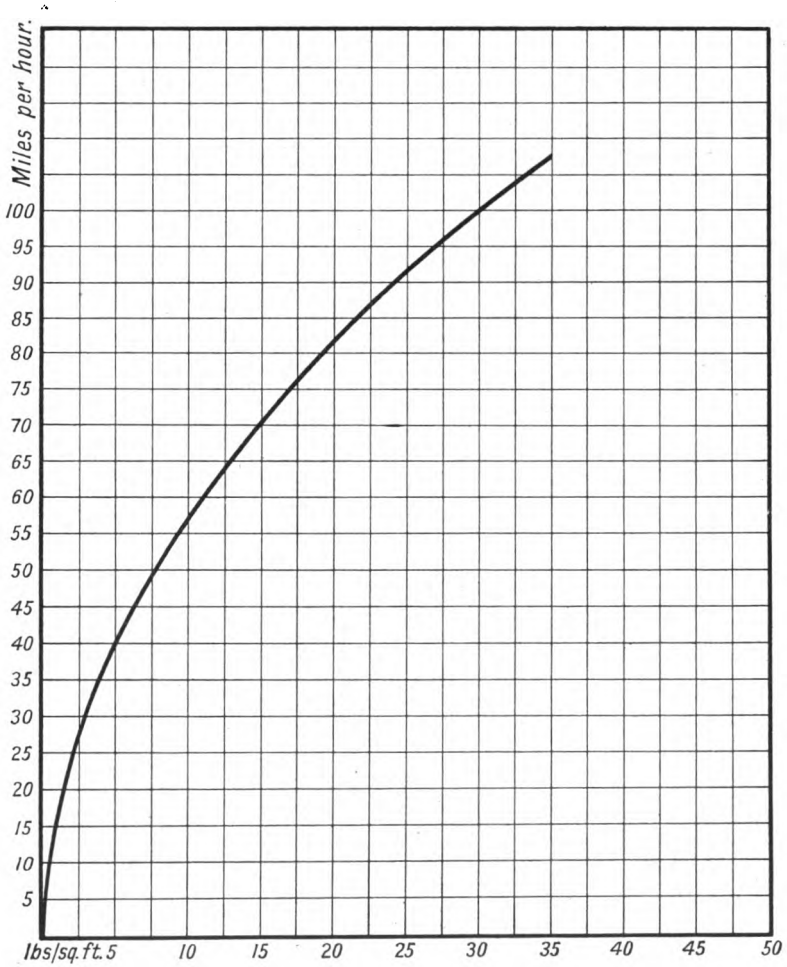
This value is practically confirmed by the experiments carried out at the National Physical Laboratory by Dr. Stanton, which, though made on very small pressure boards, were conducted with great care and skill. In these experiments a current of air was produced in a tube 2 ft. 6 in. diameter by a fan, and the velocity of the current was maintained constant throughout the section of the tube by

introducing layers of gauze at certain points. By this means a fairly uniform current was set up all across the section, the velocity of which was measured by a Pitot tube of extremely ingenious construction, for the details of which the reader is referred to a paper[1] from which these particulars are drawn. It was found that discs up to 2 ins. in diameter could be used for pressure experiments without being affected by the walls of the tube, and the plates used were, generally, not larger than this. The conclusions arrived at as a result of these observations were, that the pressure is greatest at the centre of the plate on the windward side, and diminishes towards the edges, while that on the leeward side is practically constant, being uniformly lower than the barometer, also that the pressure at the centre of the plate on the windward side is proportional to the density of the current and to the square of the velocity. It was also shown that for similar plates varying in size between the circumscribed limits of the apparatus, the mean intensity of the pressure for the same density of current was the same. For circular plates the diameter ranged from 0·5 in. to 2 ins., and this was the largest size of plate employed. Further experiments made upon large pressure boards mounted upon a steel tower show that the coefficient 0·003 is practically correct.[2]

The results, reduced to the same form as the other equations, give a coefficient of 0·0027, so that, compared

[1] The Minutes of the Proceedings of the Institution of Civil Engineers, vol. clvi., p. 78.

[2] The results of these further experiments were communicated to the Institution of Civil Engineers in a paper by Dr. Stanton read at a meeting on December 3rd, 1907.



Curve showing wind pressure according to the formula $P = 0{\cdot}003\,V^2$ *which is based upon the best experimental evidence.*

FIG. 70.

with that obtained by Mr. Dines, it will be seen to agree closely.

(1) Smeaton $P = 0{\cdot}005\ V^2$
(2) Dines $P = 0{\cdot}0029\ V^2$
(3) Stanton $P = 0{\cdot}0027\ V^2$

The unit pressure obtained from experiments on small plates is higher than that obtained for large, for the reason that, where small plates are exposed to a current of air, the pressure at the down-stream side is less than the barometer over the entire surface, while with large plates this area of low pressure is confined to a zone round the back edge of the plate. It is probable, therefore, that the actual unit pressure upon structures is much lower than that given by (3), and that with the requisite factor of safety this formula would be quite safe. Probably the correct value is not far short of 0·003, so that $P = 0{\cdot}003\ V^2$ represents the best determination in the light of our present experimental knowledge. The experiments of M. Eiffel at the great structure in Paris which he has conducted with great care for several years substantially confirm these results. Pressure plates equipped with ingenious measuring apparatus were allowed to fall from the second stage of the tower and wind velocities exceeding 40 metres per second (90 miles per hour) were recorded. The pressures calculated according to this rule are given in the fourth column of the table and are plotted in Fig. 70.

CHAPTER X.

THE APPLICATION OF WIND POWER TO INDUSTRY.

THE best available evidence upon the subject directs us to the conclusion that windmills were not used in England more than one thousand years ago, and the oldest authentic records that we have concerning the erection of a windmill bear the date of 1191. A strong testimony to the accuracy of this statement lies in the fact that the survey of mills in Domesday does not include windmills, which, had they been in existence, would have found a place in this complete record. There are however allusions by writers, long after that period, to windmills at an earlier date than 1086, but, in the opinion of two authors whose authority is unquestioned,[1] there are valid reasons for casting aside these references, and for accepting the account of the mill built by Dean Herbert in his glebe lands at Bury St. Edmunds as that of the first windmill to spread its sails aloft on English soil. Owing, however, to the fact that this mill was raised in defiance of authority, it was quickly destroyed by the order of an Abbot, and a report of the case, preserved in the Abbey, has come down to us. This mill, as indeed nearly all of those which dotted the landscape until the introduction of steam, was for grinding corn. From that time on the use of

[1] "History of Corn Milling," vol. ii. Richard Bennett and John Elton.

windmills steadily increased in England and also in France, and it is on record that King Edward III. viewed the battle of Cressy from a windmill at the top of a hill (1346). In many instances the mills in England were the property of the Lord of the Manor, who exercised "soke" privileges, by which all the grain raised by his tenants over his estate had to be milled at his mill. The first mill, according to the best accounts, was a very crude affair when compared with those of later centuries, and the only characteristic which has survived until the present day was the number of the sails which, with a few isolated exceptions, has always been four.

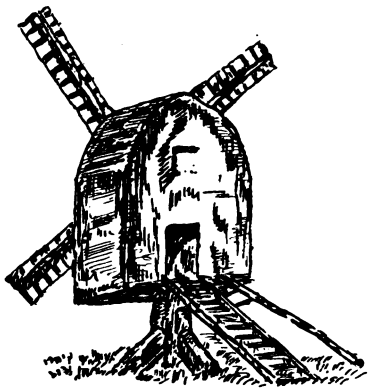


FIG. 71.—Post Mill.

In order to follow to better advantage the evolution of the windmill from the earliest forms, a few words about constructional details are necessary.

To begin with, there is the sail-shaft carrying the four sails, from which the power is transferred to the grinding buhrs through gearing. This shaft must be capable of being directed towards any point of the compass so as to present the sails to the winds from every quarter. This operation requires mechanism, which in the earlier mills was moved by hand, while later contrivances were devised to coerce the wind to accomplish the desired movement of

the sail-shaft, so that it might always lie in the direction of the wind and thus be exposed to the full wind pressure upon the sails. The earliest type of mill, which is still to be found at work in England, is shown in Fig. 71. It was probably such a type as this that a great writer discovered while on a visit to the Azores and to which he referred in these words : "Small windmills grind the corn, ten bushels a day, and there is one assistant superintendent to feed the mill, and a general superintendent to stand by and keep him from going to sleep. When the wind changes they hitch on some donkeys, and actually turn the whole upper half of the mill around until the sails are in proper position, instead of fixing the concern so that the sails could be moved instead of the mill."[1]

The wooden building containing the machinery and stones is supported upon a wooden post, upon which it is free to turn. The miller, who has to watch constantly for a shift in the wind, turns his sails accordingly by laboriously turning the mill by means of the long baulk of timber which projects outward on the opposite side to the sails. The difficulties and danger involved in milling during a gale may well be imagined when the crude nature of the earlier fabrics is considered. Often with a howling tempest raging outside, and the crashing and creaking of the machinery within, we are told that the courageous miller would, like the dauntless sea captain, stand fast to his post. In vain he would apply rude brakes to stop the whirring shaft within, while all his corn hoppers would be opened to feed the rumbling mill stones, and thereby offer as much resistance as was possible to the impetuous motion of the machinery. Sometimes his

[1] "The Innocents Abroad." Mark Twain.

attempts would prove of no avail, and with a crash his dusty habitation would be blown over, and if he were fortunate enough to escape, he would see the wreck of what was for many years his pride, pleasure, and source of livelihood. In such mills wood was the chief material used in the construction, and fire, sometimes caused by an overheated journal, was often an enemy to be reckoned with. The heavy sail-shaft, bored with two mortise holes at right angles through which the sailyards were passed, ran in rude journals, wrought-iron bands upon the shaft serving to reduce the friction to some extent. The gearing in the earlier types was that known as pin gearing, by which the necessity for nice adjustment between the gears was obviated, for the accurate alignment necessary for bevel gears was then beyond the powers of even the best millwrights of the period, and cast gears with wooden cogs were a later refinement which materially added to the efficiency of the mill by reducing the friction losses in the transmission of the power from the sails to the stones.

Chief among the improvements which contributed to the efficiency of the mill, and which relieved the miller from much misdirected attention, was an automatic apparatus for effecting the adjustment of the sails to the wind. The earlier devices for supplanting manual labour in this direction were not wholly satisfactory. One form consisted of a wide fan projecting outwards on the opposite side to the sails, upon which the wind acted obliquely until the mill was turned in the desired direction. The mill, with the machinery and sacks of corn, was so heavy and the friction so great that this plan never found extensive acceptance

in the early days, though, as will be seen later, it is now employed very generally upon the modern light steel mill. A later device consisted of a circular rack into which a pinion geared. The rack was attached to the revolving mill, which was thus turned by means of a rope and pulley similar to the apparatus used for rotating the dome in a modern observatory. But before a satisfactory change had been made in this direction the mill had undergone structural alterations, to some extent necessitated by the demand for mills of greater capacity. Sweeps or sails of 40 ft. from tip to tip were no longer the extreme limit, and 60 and even 80 ft. circles were swept by the great sails of canvas or wooden slats with which the mills were equipped. The post mill was not high enough for sails of such a size, and consequently the mill top, capable of rotation about a vertical axis, was mounted upon a short truncated cone of stone, brick, or wood, as illustrated in Fig. 72. In addition to the added stability and resistance to overturning in a gale, the basement afforded a convenient shelter and storage for sacks, and for receiving the flour as it came from the buhrs on the floor above.

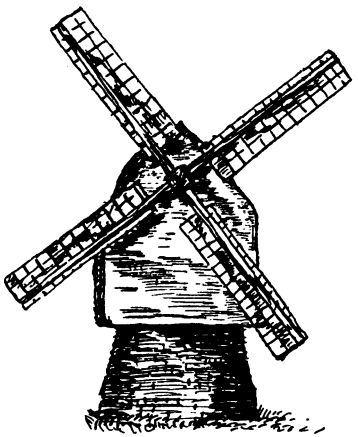


FIG. 72.—Development of Tower Mill.

It is not difficult to follow the gradual evolution of the tower mill from this primitive form. The wooden house

FIG. 73.— Tower Mill.

or top gradually assumed the form of a cap to a substantial tower of brick, stone, or wood, and was capable of rotating thereon and carrying with it the sail-shaft and connections. This process of development was slow, but during the sixteenth century the complete evolution took place, if, indeed, it is possible to indicate precisely the distinction between a tower mill so called and a mill having a somewhat enlarged rotating top placed upon a stone or brick pier. It is probable that the increasing necessity for capacity in a mill, coupled with augmented skill on the part of the millwright, were the contributory causes to a change which spread all over the country, to which the many ivy-clad remains of towers to-day bear witness.

Thus from a mill entirely constructed of wood and subject, of course, to destruction by fire, the process of development culminated in a stone or brick structure, capped by a wooden frame and roof, and which readily lent itself to the demands for size by increasing the height of the tower and the length of the sweeps. The annexed picture shows one of these mills such as may still be seen at work in many places in the country, in which the various improvements in the details of construction and machinery are exemplified, and which, while marking the highest skill of the millwright's art in the construction of windmills for corn grinding, at the same time shows the last type of those picturesque structures which the steam-driven roller mill has almost completely driven out of existence. To some extent they are still used for grinding food for cattle, but, situated as they generally are, away from lines of transport and waterways, they can never hope to compete in any sense with the gigantic roller mills located at convenient places for the

shipment of their product, and provided with a reliable and constant power.

The interior arrangement of a tower mill varies according to the size. Some of the modern examples are more than 100 ft. high, and are divided into three, four, or even five stories by floors, the ascent from one floor to the other

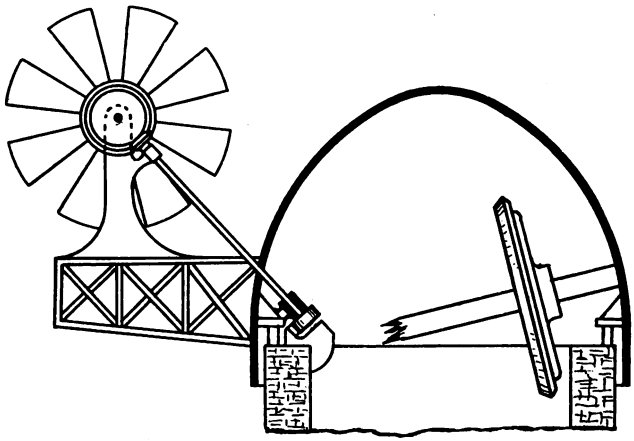


FIG. 74.—Veering Mechanism.

being made by wooden ladders. The automatic apparatus for rotating the top made it possible for the height to be increased to a great extent. In some of the larger mills an outside gallery surrounding the tower allowed access to the sails for making repairs, as they could not be reached from the ground. The mechanism of the device for automatically adjusting the sails to the wind is shown in Fig. 74. A fan-wheel is pivoted at the end of a lattice arm on the opposite side of the tower to the sails, and the plane of the fan is at right angles to that in which the sails revolve. A

bevel gear on the fan-wheel shaft meshes with a pinion, which, through a connecting shaft, rotates the pinion working in the annular rack upon the top of the tower. The top of the tower is free to move upon a series of rollers, and, as the fan-wheel is rotated by the wind, the top is moved round until the fan-wheel presents its edge to the wind and then

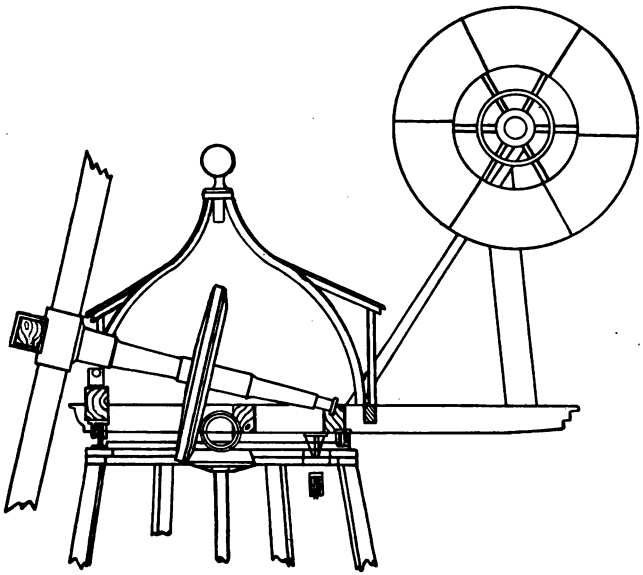


FIG. 75.—Veering Mechanism.

the sails are athwart the wind. With large and heavy mills there are two fan-wheels to render the self-adjustment more sensitive, but generally one is sufficient to keep the mill within a few points of the desired direction. Fig. 75 shows another form of this arrangement. This was, perhaps, the most important invention in the art of wind-mill construction, but was of comparatively recent origin,

and therefore has had a correspondingly short career of usefulness. Another plan, which, however, never was adopted extensively, also made use of the fan-wheel, but instead of being geared to the top of the tower it actuated a roller which was fixed at the end of a long projecting arm similar to that used in the post-mill for gyrating the entire mill about the central post. This roller moved over a track in a circular orbit, and the mill top to which the beam was fastened was thus rotated. It was at the best a rude and clumsy makeshift, and is only interesting as a link in the stages of development of the mill from the one in which manual labour alone accomplished the exacting and often dangerous task of keeping the sails full to the breeze.

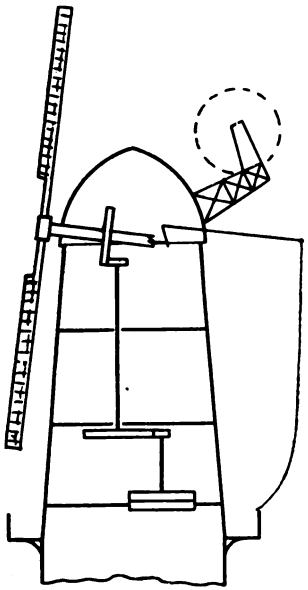


FIG. 76. — Gearing for Mill with Single Pair of Stones.

The "wind-shaft" or "round-beam," to give it the names conceded to it by millers, is in either case misnamed, for it is sheltered from the wind, and also usually has a square, hexagonal, or octagonal section; in short, many sections, but circular. Sometimes these shafts were cut from a single piece of timber, but more often they were built up in layers with broken joints, and could thus be made up of short pieces of sound beech, oak, or pine. The main gear and

brake wheel, also of wood, was divided into sections, which were held together by bolts. This gear meshed into a small pinion, called a wallower, on a vertical shaft, which also carried a large gear at the bottom end, and this in turn

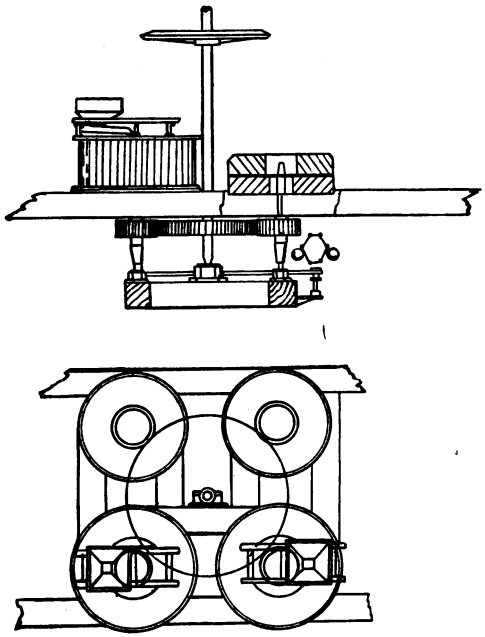


FIG. 77.—Gearing for Four Pairs of Stones.

meshed with a small pinion or "nut" on the top of the stone shaft. In some mills the top stone was driven, in others the nether stone. The arrangement of the machinery varied, of course, according to circumstances, and Fig. 76 is therefore only generally representative of the plan for a mill with a single pair of stones, and Fig. 77 shows how four pairs of stones may be driven from a single vertical

shaft. The gear ratio at the sail-shaft was about 3 to 1, and at the stone shaft 4 to 1, so that the stone makes 12 turns to one of the wind-shaft. Most mills had two sets of stones, and others had edge runners for the grinding. The edge runners consisted of two large stones at the ends of a short horizontal shaft which was rotated in a horizontal plane by a vertical shaft through the centre. The stones rolled in a trough or pan, in which the raw material was placed, and was discharged through an opening when sufficiently pulverised. The amount of material that could be passed through a mill necessarily depended upon the wind and consequent speed of the stones. The authors before referred to quote the performance of a post mill with a length of sailyard of 50 to 60 ft. from tip to tip. With a steady wind this mill would grind about twenty quarters of grist for cattle per working day, and on a very windy day twenty-four quarters of 480 lbs. (5·14 tons); and other mills of the same kind had a capacity of as much as twenty bushels of corn per hour. As the weight of contents of a bushel varies with the kind of grain, it is difficult to make correct comparisons between the performance of different mills.[1] Another post mill with a fairly steady wind is credited with six bushels of 60 lbs. per hour with one pair of stones, which would

[1] The Imperial bushel contains 2,219·28 cu. ins., or 80·18 lbs. of water at 60° F. and is equal to 36·368 litres = 1·032 Winchester bushels.

The Winchester bushel contains 2,150·42 cu. ins., being the volume of a cylinder 18·5 in. internal diameter and 8 in. deep, and this is the standard in the United States. It = 60 lbs. of wheat, = 56 lbs. of corn or rye, = 48 lbs. of barley, = 32 lbs. of oats, = 60 lbs. of peas, = 35·24 litres, = 0·969 Imperial bushels, = 77·69 lbs. of water.

appear to be a fairly correct estimate of the performance of such mills. The output of tower mills was only limited by the size of the sailyards. With one pair of stones 500 lbs. of grist per hour would be the output for a steady wind, but in a gale this might be doubled, while with a wind velocity of 13 miles an hour and sails making 11 to 12 revolutions per minute, 900 to 950 lbs. per hour was obtainable. The absence of exact data as to the spread of sail, velocity of wind, and other conditions entering into the problem render the figures of output somewhat indefinite, and the condition of the stones also invests an estimate of the efficiency of the mill with uncertainty. As the mills were classified according to output in grist, the engineer of to-day, whose ideas of mechanical efficiency are of a finer order, cannot rightly appraise the value of a train of mechanism upon which a gust of wind acts on one end and at the other the useful work is measured in bushels of flour.

For this reason, and also because of our inexact knowledge of the action of the wind of a given velocity upon a revolving sail, the mechanical efficiency of the corn-grinding windmill is unknown. If the pressure component in the plane of rotation acting on the sails at all points of the area exposed to the wind was known, it would only be necessary to multiply the pressure per unit of area by the velocity at which that part of the sail was moving to obtain the power put into the sail by the wind at that point, and the integral of these for the entire sail area would disclose the total power applied in torsion to the wind-shaft; but attempts of such a kind to arrive at the power would be untrustworthy.

Haswell gives an empirical formula for estimating the power of a well-designed mill, which the writer has checked by reference to estimates made of the power of mills by practical millers. In every case where the data was sufficient for comparison the formula has proved fairly trustworthy. If A be the total sail area in square feet and V the velocity of the wind in feet per second, the horse-power is given by the expression

$$H.\text{-}P. = \frac{A\ V^3}{1{,}080{,}000}.$$

In one case a mill with four sails, each 24 ft. in length and 6 ft. in breadth, was estimated at 4 h.-p. with a wind velocity of 20 ft. per second (13·6 miles per hour). By applying the formula we have

$$H.\text{-}P. = \frac{24 \times 6 \times 4 \times (20)^3}{1{,}080{,}000} = 4{\cdot}27.$$

The agreement in this particular instance is as close as could be expected, but in any case the formula must only be regarded as a rough guide and has no pretensions to accuracy, as it takes no account of weathering, length of sail, and the proportion of length to breadth of sail, all of which affect the power of the mill. As we have seen that the pressure of the wind upon a plane surface varies as the square of the velocity, the power developed must therefore vary as the cube, assuming that the speed of the sail is proportional to that of the wind. We have every evidence that this assumption is the correct one for plane sails as well as for the cups of the Robinson anemometer, which move with a speed proportional to that of the wind which acts upon them. It is not surprising, therefore, that in consequence

of the very rapid falling off in the power developed at the wind-shaft when the wind drops by a few miles per hour, the requirements of the miller in the matter of wind are not such as can be satisfied by the weather every day. In the case before referred to, if the wind should drop from 13·6 to 10 miles per hour, the power would fall from 4·27 h.-p. to little more than 1·68 h.-p. A wind of 8 miles per hour would not yield 1 h.-p., which would probably be insufficient for grinding even if the torque on the shaft at starting was sufficient to overcome the statical friction in the machinery and journals. Thus it was that for many days in the year the miller was forced to wait patiently for the wind to stir his sails while the sacks of corn accumulated on the mill floor. At length it would come, perhaps more than he desired, and for days and nights without ceasing the corn was converted into flour by the busy millstones, and his anxiety to make the most of that which the weather had brought him left him only time for an occasional nap throughout the long night spent amid the dust and whirr of the mill.

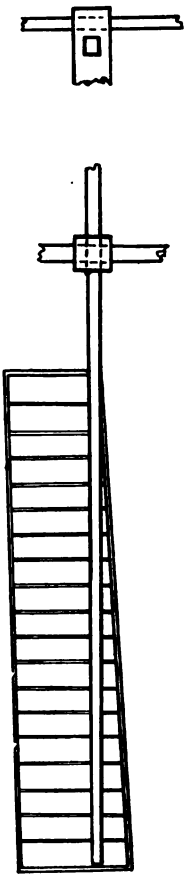


FIG. 78.—Sail as used on Tower Mills.

If the wind increased sufficiently to become dangerous the mill was stopped by the application of the brake, or

the top was turned round out of the wind, and then the hazardous operation of reefing the sails was begun. On the smaller mills the sails were reached from the ground, but the tower mills, as shown in the illustration on p. 246, were provided with a balcony from which the miller could ascend the sail-arm and take in canvas in the manner of a sailor at the yard-arm. The sails of the more modern mills were constructed of wooden slats instead of cloth (Fig. 78), and the furling was at a later date accomplished by a train of mechanism from the interior of the mill, by which the weathering of the slats was altered. This mechanism was essentially the same as that adopted and still used on certain of the light mills of to-day to be described later. The chief advantage of the cloth sail was its comparative lightness, which was no small consideration in view of the necessity of losing as little power as possible. The disadvantage, however, of the time lost in furling led to the adoption of the other principle, and eventually to the almost complete suppression of the cloth sail. In some of the mills equipped with the wooden sail the slats are laid parallel to the sail length and are simultaneously turned through the same angle in the manner of a venetian blind. This method was the invention of M. Berton, a French engineer. They are also arranged in modern mills transversely to the sail, especially when the sail-arms are very long, as otherwise they would be unmanageable. Moreover, the weathering of the sail at different distances from the centre is more accurately attained when the slats are athwart the sail length, as each member may be weathered more correctly for the radius at which it is placed. This may be understood by imagining that a venetian blind is

twisted out of the plane it would occupy if hanging free. The bars each suffer an angular deflection which may be supposed to increase towards the bottom, so that the last bar makes a considerable angle with its original position, while those near the top are but slightly changed in position. The bottom bar occupies a relative position analogous to that part of the sail nearest the wind shaft, while the top bar is at the extreme tip of the sail. To carry the analogy further, the reefing is represented by operating upon the hanging blind so as to change the angles which the bars make with a horizontal plane, such as by pulling upon the tapes with which venetian blinds are usually provided.

The wind shaft is set at an angle to the horizontal, which varies according to local conditions from 3° to as much as 35°. There appears to be no other reason for this than that of allowing the sails to revolve clear of the tower, which usually has a batter of about 1 in 7, or 1 in 8, though it has also been ascribed to a supposed tendency of the wind to blow at an angle with the horizontal in a downward direction when on low ground. It is usual to make the sails rectangular with a ratio of length to width of about 5 to 1, the whole sail surface not exceeding one-fourth of the circle swept by the sail arms. The rectangular sail is in some mills supplemented by a triangular addition upon which canvas is stretched also, as illustrated in Fig. 78.

As it is doubtful if any of the readers of these pages will ever be called upon to design a windmill of a type which is so rapidly disappearing, it will not be advisable to take up much space in describing how the proportions of sails and "weathering" (angle which the sail makes with the plane

of revolution) were arrived at. Even if such a course were desirable, the information at hand is very scant and is of the rule-of-thumb kind. Smeaton left us a set of rules which were the outcome of a series of experiments, the results of which he communicated to the Royal Society in 1759. Of these results the most important related to the angles which he pronounced to be the best for the weathering of the sail. His results are as follows:—

Distance from centre of motion, 1, 2, 3, 4, 5, 6.

Angle with plane of motion, 18°, 19°, 18°, 16°, 12·5°, 7°.

Thus if the sail arm be divided into six equal parts, the angles which the sail makes with the plane of motion on the division lines are set forth, the angle decreasing towards the extremity of the sail until at the tip it is 7°. He also laid down the rule that the best speed for the tip of the sails should be 2·6 times the velocity of the wind, which implies that a point on the sail located at 38·5 per cent. of the radial distance moves at the same speed as the wind. Other angles have been used, which in some mills become almost 0° at the tip of the sail. The following angles are elsewhere given as an alternative:—

Distance from centre of motion, 1, 2, 3, 4, 5, 6.

Angle with plane of motion, 24°, 21°, 18°, 14°, 9°, 3°.

The general conclusions to which Smeaton was guided by his experiments with wind velocities of 2·7 to 6·1 miles per hour are as given below. It is to be observed that these wind velocities would be regarded as too low for the practical operation of windmills, as a wind of 10 ft. per second (6·8 miles per hour) was generally insufficient to drive a loaded mill, while 35 ft. per second (23·8 miles per hour) was considered too high, and precautions had then to be taken by

reefing sail to guard the safety of the structure in a gale of this severity.

1. The velocity of windmill sails, so as to produce a maximum effect, is nearly as the velocity of the wind, their shape and position being the same.

2. The load at the maximum is nearly as, but somewhat less than, the square of the velocity of the wind, the shape and position of the sails being the same.

3. The effects of the same sails, at a maximum, are nearly as, but somewhat less than, the cubes of the velocity of the wind.

4. The load of the same sails, at the maximum, is nearly as the squares, and their effect as the cubes of their number of turns in a given time.

5. When sails are loaded so as to produce a maximum effect at a given velocity, and the velocity of the wind increases, the load continuing the same—first, the increase of effect when the increase of the velocity of the wind is small will be nearly as the squares of those velocities; secondly, when the velocity of the wind is double, the effects will be nearly as 10 to 27·5; but, thirdly, when the velocities compared are more than double of that when the given load produces a maximum, the effects increase nearly in the simple ratio of the velocity of the wind.

6. In sails where the figure and position are similar, and the velocity of the wind the same, the number of revolutions in a given time will be reciprocally as the radius or length of the sail.

7. The load, at a maximum, which sails of a similar figure and position will overcome at a given distance from the centre of motion will be as the cube of the radius.

8. The effects of sails of similar figure and position are as the square of the radius.

9. The velocity of the extremities of Dutch sails, as well as of the enlarged sails, in all their usual positions when unloaded, or even loaded to a maximum, is considerably greater than that of the wind.

CHAPTER XI.

MODERN WINDMILLS—CONSTRUCTIONAL DETAILS.

The Modern Windmill.

The subject of this chapter is a type of mill somewhat different in construction and working from those which we have just considered. In a sense it cannot be said to be a replacement of the wooden mills, which are still used in rural districts for corn grinding, but it has a wider future and range of utility for pumping water and for driving light farming machinery and, to a very limited extent, for the generation of electricity. Indeed, so widely used are these mills in great agricultural countries like Australia and the western parts of America and the Argentine that the manufacturers have found difficulty in keeping pace with the demand, notwithstanding the fact that there are several firms that turn them out by the thousand, and who can always find a ready market for them. They are cheap and are well made, will work without much attention, and are admirably adapted to pumping water and to any service not requiring a continuous output of power. They are easily erected, require very light foundations, and repairs and renewals are a small item, as they are now made capable of withstanding the heaviest gales.

In appearance they are strangely unlike the old mill. The heavy sweeps, sometimes 30 ft. or 40 ft. long, give place

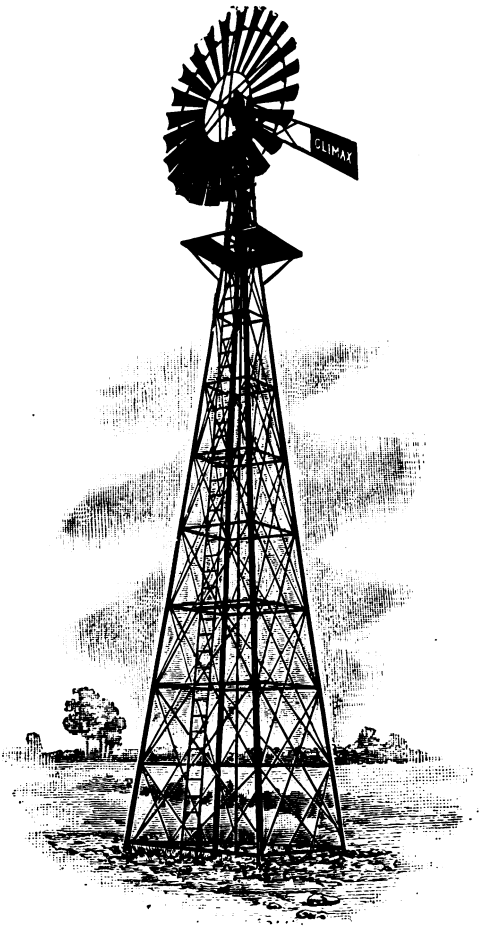


FIG. 79.—Steel Windmill and Tower.

to a light disc made up of slats, not unlike the sections of a fan, and which is, in the largest mills, not greater than 40 ft. in diameter, while the vast majority of these mills that are sold are not greater than 10 ft. to 12 ft. in diameter. Indeed the writer was informed by a leading manufacturer that the 10 ft. or 12 ft. mill is likely to be the standard size, and that a manufacturer who can turn these out well made and cheap has always a ready market for them.

These mills are erected upon steel towers, which may be constructed to a height above ground of as much as 80 ft., or else they may be mounted on a house-top with a low tower sufficient to provide the necessary clearance from surrounding obstructions, so that the sails may be fully exposed to a wind from any quarter of the compass. Fig. 79 is a typical example of a mill supported upon a steel tower built up of rolled shapes and braced diagonally by light tie-rods, so as to form a rigid structure. A ladder is provided for access to the platform at the mill, and the pump rod may be seen in the centre of the tower. Sometimes the tower is also the support for a water tank, in which case it is constructed somewhat heavier than if it is intended only to bear the weight of the mill and pressure of the wind.

Where timber is cheap and good, wooden towers may be erected, but generally steel towers are the best for English mills, as there is but little difference in the cost, when the longer life of a steel tower is taken into account. The size of timbers used in the construction of a tower vary according to the size of mill. The Aeromotor Company, of Chicago, gives the following sizes for the construction of the head of the tower, where the four corner-posts come together, and to which a short steel head is attached.

DETAILS FOR TOPS OF WOODEN TOWERS TO WHICH SHORT STEEL STUB TOWERS ARE ATTACHED.

SIZE OF MILL.	SIZE OF TIMBERS, INCHES.
8 and 10 ft. .	4 × 4
12 ft. . . .	4 × 4 or 4 × 6
14 ft. . . .	4 × 6 or 6 × 6
16 ft. . . .	6 × 6, 6 × 8 or 8 × 8
20 ft. . . .	8 × 8 or 10 × 10

Where more than one size of timber is shown, the small sizes may be used if the timber is free from knots and shakes, and has a straight grain; otherwise the second size is expedient and should always be used on the lower parts of towers when they are higher than 40 ft.

The batter for wooden towers should be somewhat greater than for steel, and ought to be more than one-fifth the height. The Aeromotor Company recommends that the posts should be slightly sprung to an increasing batter and that they should be assembled with bolts, and not with screws or spikes, which are liable to work loose under the vibration. Attention should be directed to the bolts from time to time to ensure their tightness, and especially for the first few months succeeding erection, as the members are liable to be sprung and looseness developed at the joints. To prevent rot at the base of the tower it is necessary that the wooden anchors should be painted with a coating of hot pitch or asphaltum, and that the tower itself should receive a coat of paint composed of boiled

linseed oil and white lead with the colour added, at least every second year. In the erection of wooden towers they may be assembled in a horizontal position on the ground and then raised to the vertical by means of blocks and tackle, and the mill afterwards hoisted into place. Heavy towers may be built up in a standing position.

The foundations necessary for the tower, whether of wood or steel, are best constructed of concrete, in which the posts are embedded. The best proportions for the concrete are two parts of Portland cement, three parts of sand, and five parts of broken stone, both sand and stone being clean and sharp. After these ingredients are mixed dry by turning over and over, the water is added and the mass tamped into the form prepared for it. If the foundation is made in stiff earth wooden forms are unnecessary, and square holes dug to a depth sufficient to sink the foundation bolts are all sufficient. It is advisable to protect the top of the foundation bolts by long wooden casings, some 2 ins. square, which can be withdrawn when the concrete begins to harden. This allows the bolts to be moved laterally so that they may be correctly placed, and the space round the bolt can afterwards be filled with grout. The correct position of the bolts may be assured by a wooden template with holes through which their upper ends are passed.

The steel towers are built up usually of light angle bars and, for the smaller mills, have sometimes only three legs, the ground plan being an equilateral triangle. The sections are fastened up with bolts, and are usually fitted together in the shop previous to shipment, so that when on the ground they may be quickly erected. In the large windmill factories the pieces are cut and punched to

templates so that they are interchangeable, and a tower of a certain height is built up of members of standard sizes. After erection they receive a coat or two of paint, and are then ready for mounting the windmill.

The wheels of the American mills are constructed of galvanised iron or of wood. Fig. 80 shows a metal wheel with curved sails supported by a circular band through

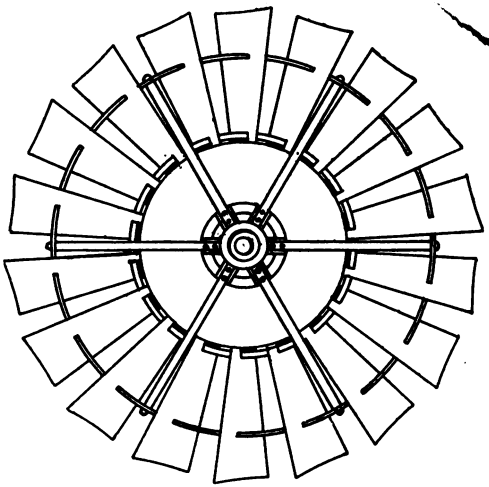


FIG. 80.—Wind Wheel of Modern Mill.

their centres, and by another at their inner extremi[ties] and connected with the centre by six arms. The sails [are] riveted to the supporting pieces as it has been f[ound] difficult to keep bolts tight. Some makers construct [the] wheels of ash or maple, the slats being bolted to the [arms] and girts, and the tendency of the wind pressure to coll[apse] the wheel is prevented by extending the axle and attach[ing] guy rods by means of a star-shaped casting, which [is] fastened to the wheel at a radius from the centre.

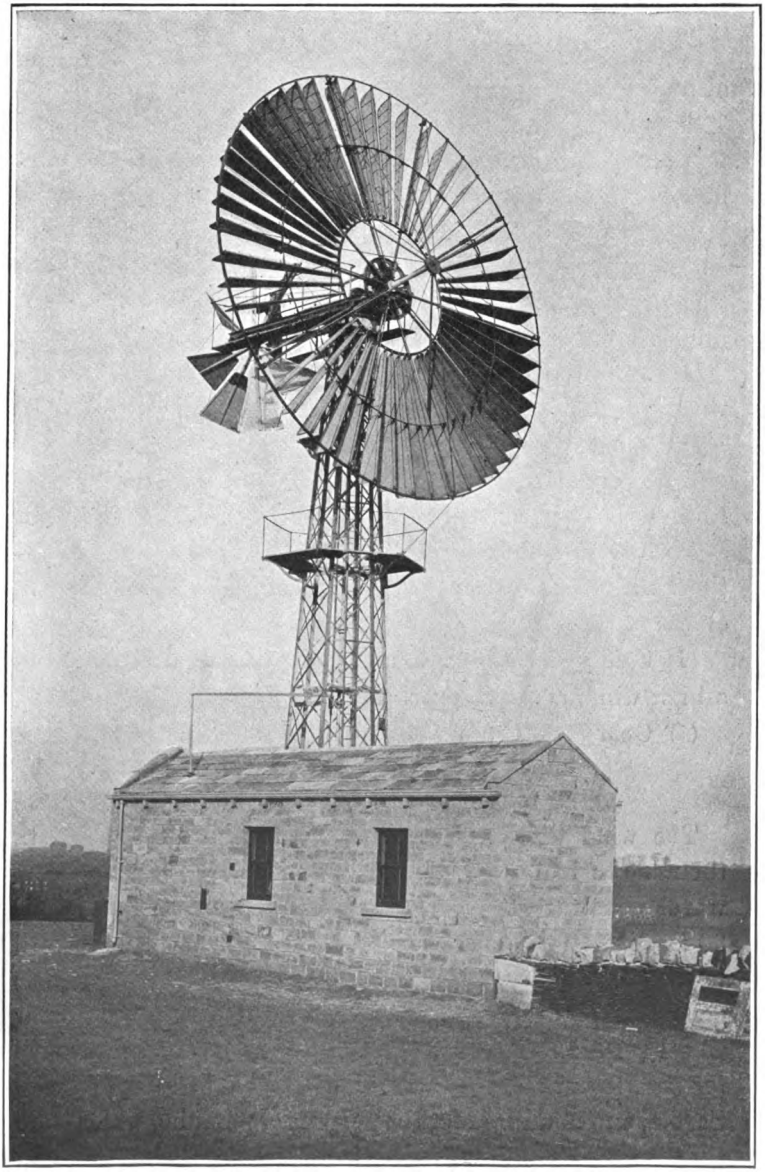


FIG. 81.—Windmill (30 ft. diam.) connected to Electric Generator.

also used in metal wheels and are shown in the illustration. This makes a very strong construction, as the wind pressure is taken up by tension in the guy rods, which may be of small section, and by compression in the axle. Indeed these wheels, if presented broadside to the heaviest gales, will not suffer injury when constructed carefully, though the safety devices attached to most of these modern mills prevent them from being subjected to the severest tests.

The essential parts of a windmill of this type, and which will be dealt with in the following order, are :—

(1) The wheel proper.

(2) The method of keeping the wheel abreast to the wind from whatever quarter it may blow.

(3) The transmission gear for transferring the motion of the wheel to the pump or other machine it is required to drive.

(4) Safety devices for averting damage or destruction in a high wind, and for governing.

(5) Gear for stopping and starting.

The Wheel Proper.

The wind wheel ranges in size from 6 ft. to 40 ft., though the latter is an exceptional size. Fig. 81 shows a wheel constructed by Mr. Titt, of Warminster, which is 30 ft. diameter, and is mounted upon an hexagonal steel tower 35 ft. high. This wheel is used to drive an electric generator through gearing. Fig. 82 shows an even larger wheel (37·5 ft. diameter) by the same maker, constructed for the Italian Government. It is erected at Margherita di Savoia, and is utilised for the purpose of pumping water from the sea for distribution in vaporising beds for the reclamation

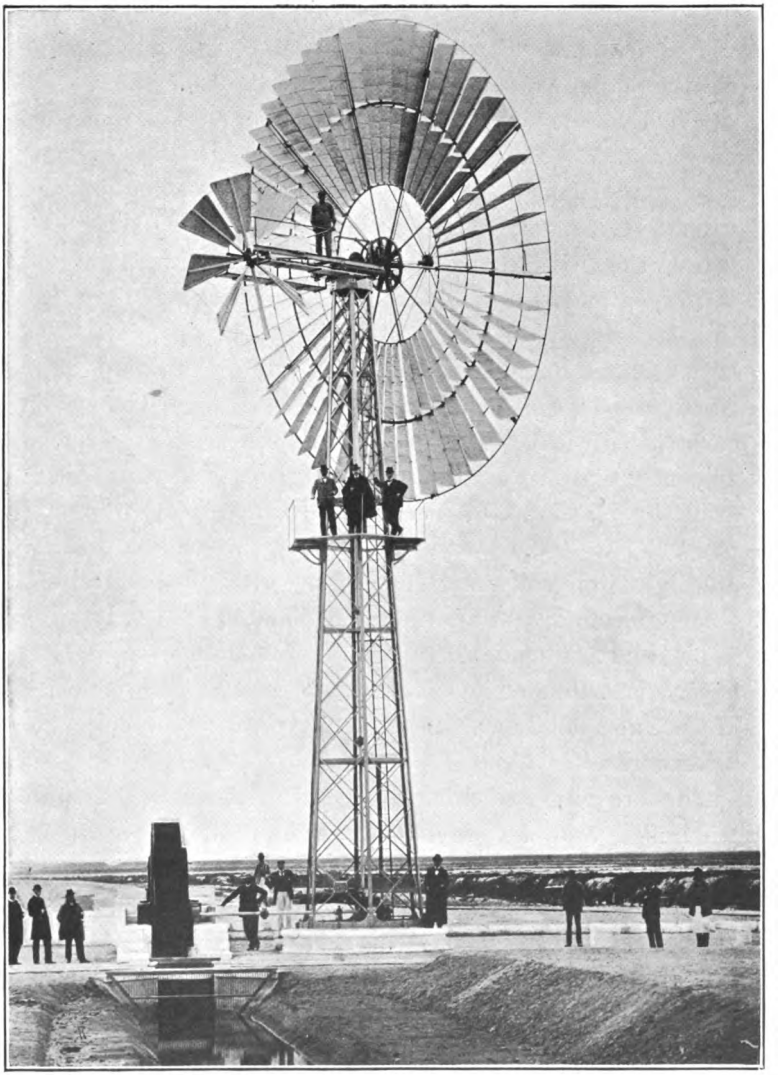


FIG. 82.—Windmill (37·5 ft. diam.) employed for Pumping.

of the salt. The dash wheel pump is worked through bevel gearing, which may be seen in the illustration.

This wheel has 100 sails arranged in two concentric rings of fifty each. The sails are plane surfaces and are hinged about their front elements so as to be turned at any angle to the wind. When they are normal to the plane of the wheel the wind passes through without imparting a turning moment and the wheel is motionless. In the illustration they are shown in this position.

In another installation by the Warminster firm, at Bury St. Edmunds, the wind wheel is 40 ft. in diameter, and is constructed with angle iron rims connected with the centre spider by ten tubular arms. It is fitted with fifty sails, each 12 ft. long and 2 ft. 3 ins. wide at the extremity, tapering down to 1 ft. These sails are made with hard wood backs and light iron rims, and are covered with oiled sail cloth. They are connected with the main wheel by ball and socket joints, and are regulated through a system of levers. This engine is estimated to give 8 h.-p., with a wind velocity of 16 miles an hour. It is employed for pumping at a waterworks.

The foregoing are unusually large specimens of modern windmills, and for such sizes light material must be employed in the construction of the sails, as metal would be too heavy, though probably aluminium would be efficient were it not for the present high price of the metal. The risk of destruction in a gale consequent upon the difficulty of stiffening a large wheel has led to a reduction in the size of the wheel, coupled with a change in the material from wood and canvas to galvanised sheet iron or steel, which is the most extensively used material out of which

sails are constructed. The mills used throughout the great agricultural countries are constructed of galvanised iron or steel and, as before stated, are for the most part about 10 or 12 ft. in diameter.

The metal sail lends itself more readily to shaping than a wooden or canvas sail, and as a rule such sails are curved more or less like a screw propeller, so that the surface of the sail makes an angle with the plane of the wheel which is less as the distance from the centre is increased. In Fig. 83 the angle θ is the inclination of the line joining the edges of the sail with the plane of the wheel, and A divided

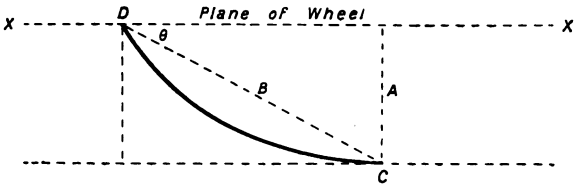


FIG. 83.—Curvature of Metallic Sail.

by B (sin θ) becomes less as the radial distance outward is increased. In an actual case the values of this ratio at the inner and outer extremities of the sail was such as to make the angles 39° and 34° respectively, the intermediate points having angles between these two extremes.

The correct curvature is more a matter of trial than of deduction from any known law or rule, and while there may be a correct curvature on a sail for a wind of given velocity, an alteration in the velocity would require a corresponding change in the curvature, so that, with fixed sails, the angles at which they are set is, at best, a compromise. But, besides having the helical curvature, all metal

sails are dished so as to present a concave surface to the wind. There is no reason for this except to add stiffness and convey strength to the sail, so that, supported as they are in some wheels only at their extremities, they may be able to resist the collapsing tendency of the wind pressure. The dished sail cannot strictly follow a helical surface, as different elements, at the same distance from the axis, are inclined at varying angles with the plane of the wheel, and only in so far as the line *CD* (Fig. 83) is inclined at an angle with the plane of the wheel, which would be correct for a helical surface of given pitch, can it be said that windmill sails are designed in accordance with any theoretical assumption; for the rest, it is guesswork supported by such experimental evidence of the behaviour of mills in winds that is to be had. Experiments have shown that there is only a slight advantage in twisted sails over sails which have a constant angle to the plane of the wheel at all points. The rear edge of the sail should be nearly parallel to the plane of the wheel, while the forward edge should be at such an angle that the wind will enter parallel to the tangent, but by twisting the sail so as to effect the proper entrance of the wind, the rear edges from which the wind escapes are placed less advantageously, which reduces the benefits derived from a twisted sail.

The best angle of weather, *i.e.*, the angle that the sail makes with the plane of the wheel, has been determined for wheels in which the sails are made of plane slats. This angle ranges from 25° to 40°, depending upon the width of the sail and diameter of the wheel. As, however, the angle for metal wheels varies with the distance from the centre, the angle of weather cannot be stated exactly, but

at the mean radius it is not far short of 36°. With fixed plane sails of wood or canvas it is usually smaller, and 27° may be taken as a fair average. If a constant velocity of wind were assured the correct angle could be accurately determined, but nature does not assist in this respect towards the solution of the problem. The action of the wind upon the sails of a wheel is but little understood, which is not surprising considering the complex movements of currents of air. The energy contained in a moving stream of air is calculated in the same manner as that employed in ascertaining the energy in a current of water. Thus, if W be the weight of air passing a given point per second and v the velocity, the energy which the air current is capable of yielding up per second were it brought to rest would be

$$\frac{1}{2}\frac{W}{g}v^2.$$

If γ is the weight of a unit volume of air at a given temperature and barometric pressure, then $W = \gamma v A$, where A is the cross-sectional area of the current. By substituting this value for W we find that the energy yielded up per second is proportional to the cube of the velocity, or the power is expressed by

$$P = \frac{1\gamma}{2\,g}Av^3.$$

It is possible by means of this formula to determine the efficiency of a windmill, for, if the constants are known, and the power exerted by a wheel be measured by a brake, the ratio of the actual power exerted to the total power gives it directly. For example, a wheel 10 ft. in diameter

has a projected area of 78·5 sq. ft., which would be the cross-section of the stream of air intercepted by the wheel. The velocity of the wind is 12·3 miles per hour (18·1 ft. per second), so that the weight of air passing the wheel per second is: $W = 0{\cdot}08 \times 78{\cdot}5 \times 18{\cdot}1 = 114$ lbs. The energy expended in bringing this current of air to rest per second would be

$$E = \frac{1}{2} \times \frac{114}{32{\cdot}2} \times (18{\cdot}1)^2 = 580 \text{ ft.-lbs. per second}$$
$$\text{or } 1{\cdot}05 \text{ h.-p.}$$

A brake test on the wheel showed that, when working at the speed which gave the maximum power, energy was given up at the rate of 84 ft.-lbs. per second. The efficiency would be therefore

$$\frac{84}{580} = 0{\cdot}145 \text{ or } 14{\cdot}5 \text{ per cent.}$$

A large proportion of the air passes through the interstices between the sails without reduction of velocity, and that which impinges upon the vanes is only partially reduced, so that of the total energy in the air a small percentage actually is turned into useful work, part of which is lost in friction in the bearings of the wheel.

Veering Mechanism.

Various methods were devised for keeping the wind wheel abreast to the wind in the old types of mill. In the light metal mill there is only one device now employed—that of a tail vane upon an extended arm, upon which the wind acts until it causes it to rest in a position with its plane parallel to the direction of the wind. In so turning,

the top of the mill, carrying wind wheel and gearing, is also moved round so that the wheel is set with its plane at right angles to the wind. This vane is shown in Fig. 1, which is representative of the type universally employed. Some makers hinge the tail vane arm so that when it is desired to stop the mill it may be drawn round by lanyards until the plane of the vane is in the plane of the wheel, and thus the wheel presents its edge to the wind. The tail vanes are made of sheet metal, and are supported by steel or iron rods riveted to them and to the head of the mill. The correct size for these vanes is derived from experience. If made too large they keep the mill perpetually oscillating in a high wind, and if too small the mill does not respond to slight changes in the direction. As the best mills are mounted upon roller bearings, the effort necessary to turn them is small, as the wind pressure on the sails is balanced unless in special cases to be referred to later.

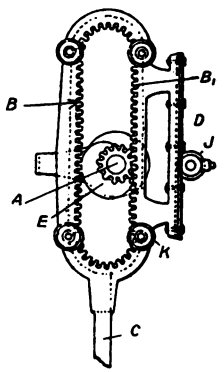


FIG. 84. — Transmission Gear on Brantford Mill.

TRANSMISSION GEAR.

The feature that most distinguishes one mill from another is the transmission gear for converting the rotary motion of the wind wheel into a reciprocating motion for the pump plungers. In the trials that have been made of wind engines this feature has been the principal cause of superiority of one type over another, for the power of the wheels is so small that transmission losses should be reduced to as fine a point as possible in order that sufficient

power may be actually employed in pumping, and notwithstanding the care taken to reduce these losses between the

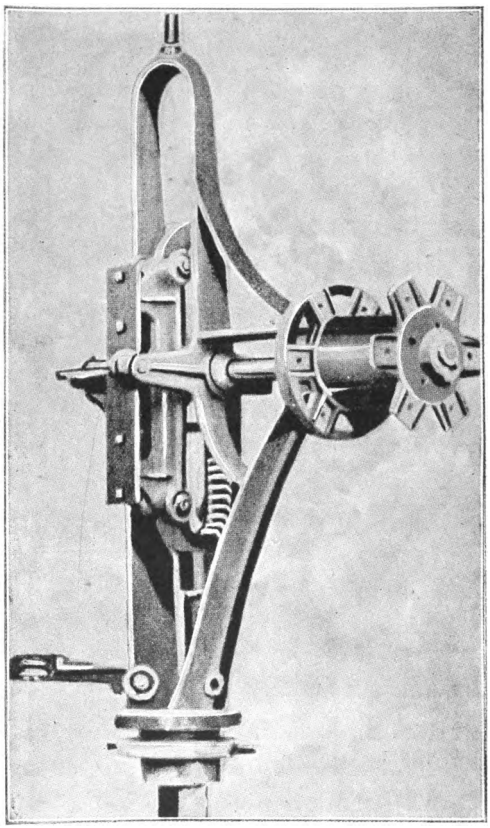


FIG. 85.—Transmission Gear on Brantford Mill.

wind wheel and pump, they are still so large, even in the best mills, as to be one of the chief determining causes of the limited value of wind engines.

For the purpose of transmission the familiar crank and connecting rod has certain disadvantages which, for this purpose, renders it objectionable. If a long stroke be desired for the pump the working of the mill is irregular owing to the leverage which the mill has to overcome in a long crank. In the Brantford mill (Messrs. Rickman & Co.) this difficulty is overcome by the mechanism

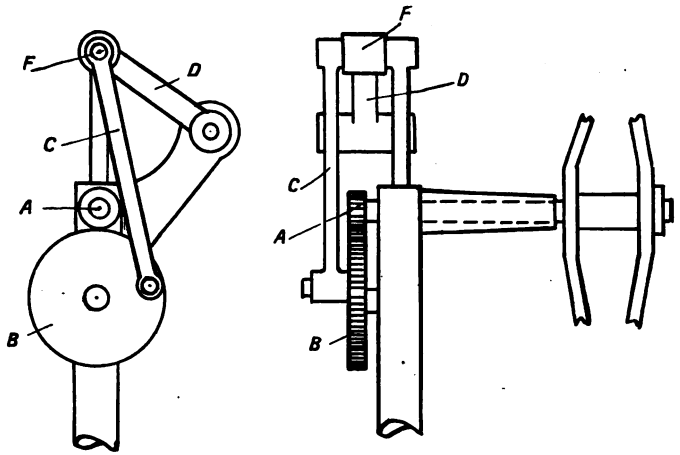


FIG. 86.—Form of Transmission Gear.

shown in Figs. 84 and 85. The sail-shaft A carries a small pinion which meshes with a mangle rack B attached to the top of the pitman C. The mangle rack is in motion alternately up and down according as the pinion engages with B or B_1. At the end of the stroke two cams E, which are also keyed on to the sail-shaft, engage the rollers K and throw the rack over so that the pinion, riding round the semi-circular racks at the ends, engages

alternately with the two straight racks. The acceleration of the pitman is thus rapid, but, instead of the harmonic motion of an ordinary connecting rod gear, a uniform speed for the pump plunger throughout the greater part of the stroke is attained. The flanged rollers J riding upon the plate D guide the mechanism and constrain the gears to mesh properly. This device has the distinct advantage of working equally well whether the stroke of the pump be 8 ins. or as much as 36 ins.; and the pump valves are rapidly opened and closed at the ends of the stroke, which helps the efficiency of the pumps materially, while with harmonic motion the operation of the valves is tardy, and in the case of large pumps has led to the use of mechanically operated valves. This

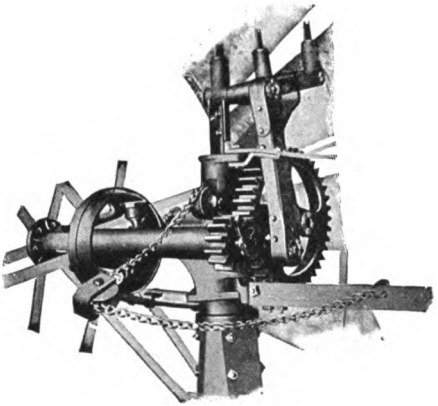


FIG. 86a.—Head of Mill showing Gearing.

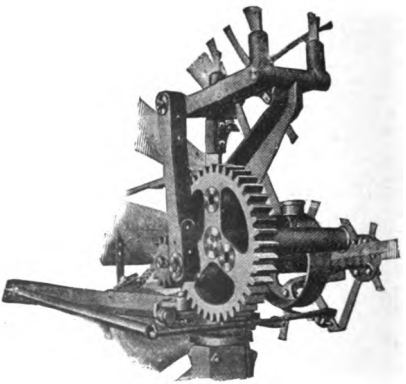


FIG. 86b.—Head of Mill showing Gearing.

is the best mechanism so far devised for converting the rotary motion of the sail shaft into a reciprocating motion.

Another form of gear is shown in Fig. 86. The sail-shaft carries a pinion A at its extremity, which meshes with a gear B giving a reduced speed, the ratio being 4 or 5 to 1. The pitman C is attached to a pin on the gear, and at the other end to a rocker arm D, swinging about a

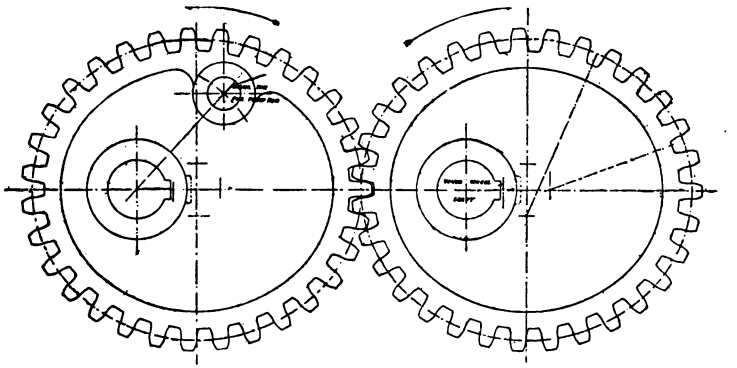


FIG. 87.—Elliptic Gear Transmission.

bearing on the main casting, and the pump rod E thus partakes of the reciprocating motion. The throw of the pump is limited in practice with this mechanism to about 18 ins., and with the exception of the fact that the motion of the pin F is not in a straight line, and that, therefore, the pitman oscillates through a small angle, the motion would be that of the ordinary crank and connecting rod. Figs. 86a and 86b illustrate this form of gearing. The gear shown in Fig. 87 consists of a pair of elliptic gears,

one of which is keyed to the wind wheel shaft, and the other to a counter shaft. The pump rod is attached by a pin to the gear on the counter shaft, and thus receives a reciprocating motion, the up stroke being slow but the down stroke rapid, owing to the ellipticity of the gears. Thus, while the pump is drawing from the well the motion

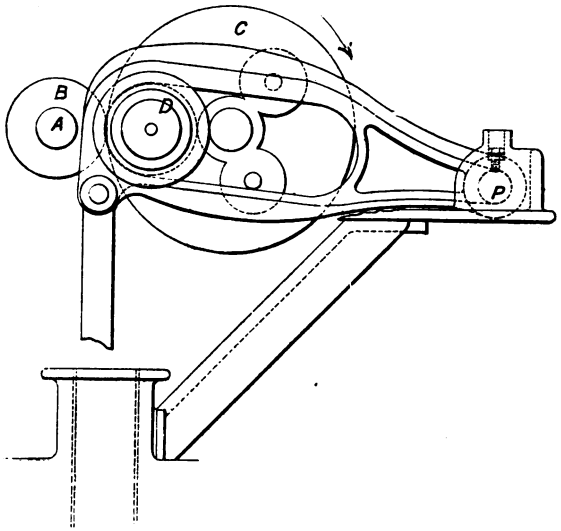


FIG. 88.—Transmission Gear.

is slow, and the descent of the plunger then takes place rapidly. The advantages of this irregular motion are doubtful, and the device is used only to a limited extent, for the utility of the quick return is lost if the pump is forcing water against pressure. Another form of quick return mechanism is shown in Fig. 88. The shaft is driven from the wind wheel shaft by a spur wheel and pinion C

and B, and the pin D, moving in the slot of the lever, gives the pitman a quick motion in descending, and a slow motion on the up stroke when the shaft is revolving in the direction shown by the arrow. The shaft has usually

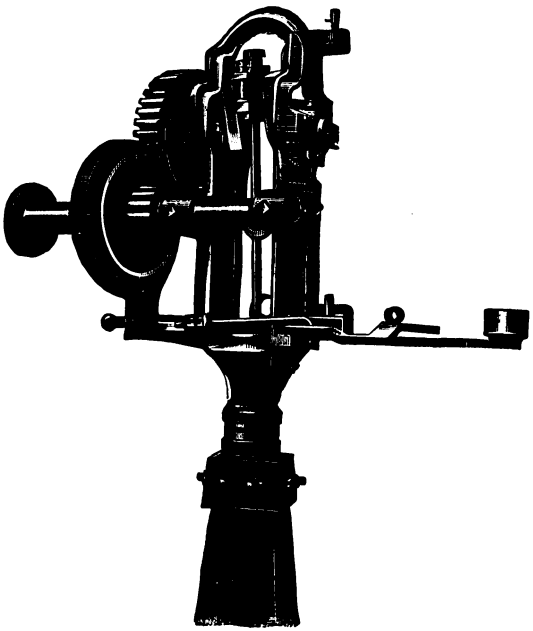


FIG. 89.—Crank and Connecting Rod Transmission Gear.

about one-third the angular velocity of the wind wheel shaft. This device is used on the Hercules mill, and similar mechanisms are employed by other makers. These motions are all open to the objection of imparting a varying resisting moment at different parts of the pump stroke, so that in light winds the mill is at a disadvantage in starting,

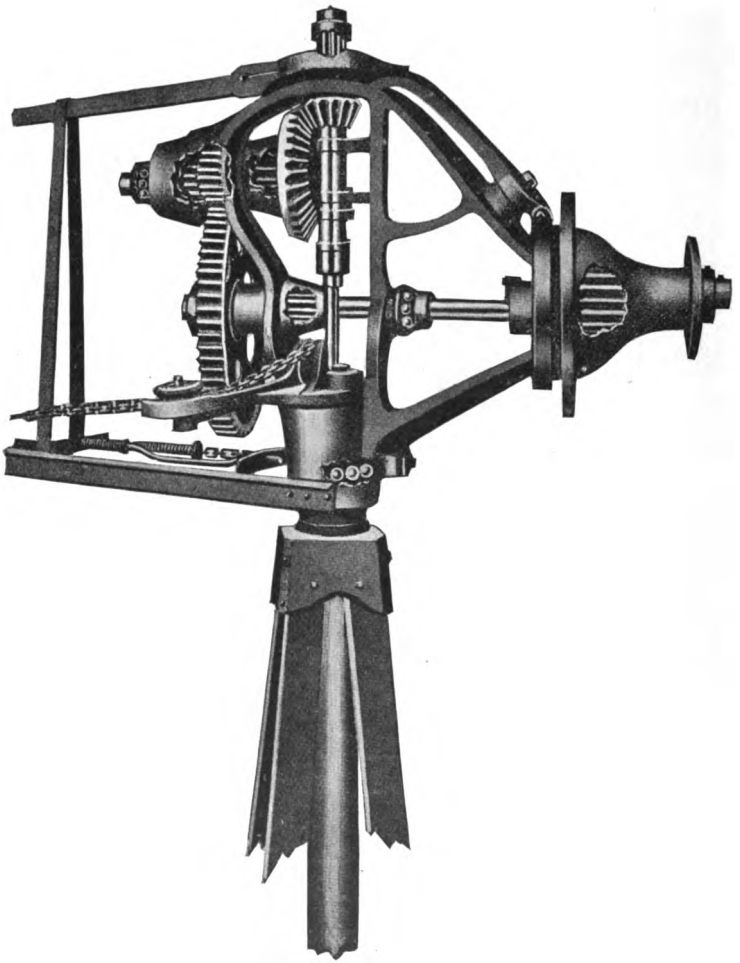


FIG. 90.—Bevel Gear Transmission.

as the pump rods and plunger weights acting at the leverage of the length of the crank have to be overcome. Gearing is necessary unless the cranks, and consequently the stroke of the pump, be made very short. The straight line motion on the Brantford mill is free from these objections, as the pinion is small, and the length of the pump stroke is independent of the size of the gearing, but the speed of the pump is effected directly by the diameter of the pinion on the sail-shaft.

Messrs. Thomas & Son, of Worcester, use a gearing on their "Climax" mills shown in Fig. 89, and the wind wheel makes 2·5 revolutions to one stroke of the pump rod. The connecting rod is a steel forging with gun metal head connection to the crank. The cross-head to which the connecting rod and pump rod are connected is fitted with rollers which travel in roll paths in the head casting. The bearings are gun metal and are lubricated by a revolving chain from an oil reservoir cored out of the casting beneath each bearing. These bearings will run for a considerable time without attention.

The mills which are employed for various other purposes about a farm, such as corn crushing, chaff cutting, etc., do not require a transmission gear of a special kind, for the rotary motion of the sail-shaft can be directly communicated to the vertical shaft through bevel gears, and in the same manner the power may be taken off the vertical shaft or by a belt pulley at the lower end. An arrangement of bevel wheel and pinion on the head gear is illustrated in Fig. 90. The bevel pinion is placed above the gear on the vertical shaft, but it may also be placed below. The arrangement shown has the advantage that the gears tend to keep in

mesh, while with the pinion below, any displacement of the vertical shaft acts to separate them. The shafts of this head gear are borne in roller bearings, and ball bearings are used to take up the thrusts on both wind shaft and vertical shaft. As the chief value of these windmills is for pumping purposes, this form of transmission is less used than those which convert the rotary into a reciprocating motion.

Safety Devices.

Safety devices for averting damage to the mill in a high wind also perform the function of governors by regulating the sail area or the exposure of the wheel to the wind according to the strength of the wind. These devices are essential on all mills, which would otherwise be quickly shaken to pieces in a heavy gale if there were not some provision for automatic adjustment. In the old mill with long wooden sweeps covered with canvas the adjustment by reefing the sail was performed by the miller, but the inconvenience of constant attendance, and the fact that the prevailing pattern of mill has fixed sails, the area of which cannot be altered to suit the wind, render recourse to some other form of regulating arrangement necessary.

The following methods of regulation are in use:—

(1) Automatic alteration of the angle of weather, *i.e.*, the angle the plane of the sail makes with the plane of the wheel, according to the strength of the wind.

(2) Automatic change in the angle that the plane of the wheel makes with the direction of the wind, so that the wheel is not always normal to the direction of the wind.

The system of regulation by altering the angle of weather has been used in mills of the type described in Chapter X., but it is not automatic. Some mills of the older types which are now used in England and elsewhere have sails made up of a series of wooden slats arranged across the width of the sail. These slats are capable of being rotated around

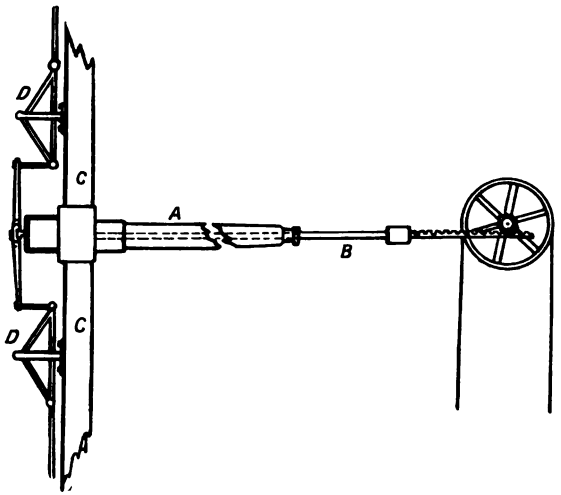


FIG. 91.—Mechanism for Weathering Sails.

an axis parallel to their long dimension, and are moved in unison by the mechanism shown in Fig. 91. The sail-shaft A is made hollow, through which the iron rod B passes, having at the rear end a rack into which a pinion meshes. By means of the linkage D the longitudinal movement of the rod B is communicated to the rods parallel with the sail arms C, and these rods are attached to the sails and rotate them all simultaneously through the same angle

when the pinion is turned by the rope wheel on the pinion shaft. The necessity for watchful attendance on the part of the miller is not obviated by this plan, but Fig. 92 shows an automatic device, which is used on an American mill, the wheel of which is composed of radial sails D hinged at

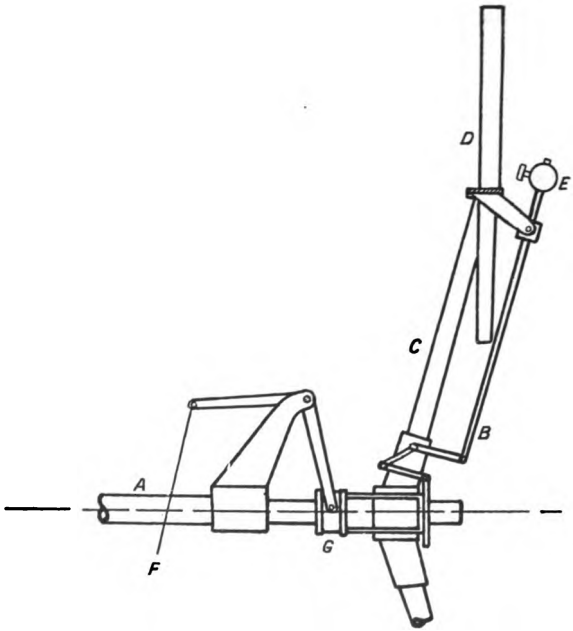


FIG. 92.—Automatic Weathering Mechanism.

the ends of radial arms C and capable of being altered in position through the linkage and rod B. The weights E act to balance the mechanism, and also to assist regulation by the action of centrifugal force upon them, which causes the sail D to assume an inclined position upon which the wind will have less effect when the velocity increases. The

regulation is at the same time effected by hand through the cord F and the bell-crank lever which moves the collar G upon the sail-shaft.

This method of regulation is only of course adapted to mills with movable sails, and is therefore of very limited application. In fact it is confined to a class of mill which is numerically very much less used than the steel mill

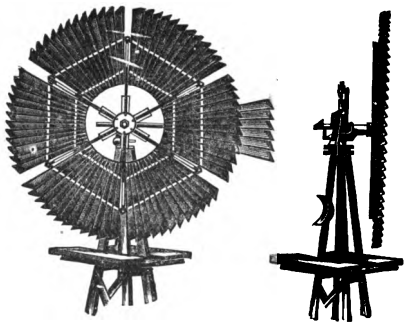


FIG. 93.—Governing Mechanism (Eclipse Mill).

with fixed sails, and other systems of regulation have therefore come into use.

One form of governing mechanism is shown in Fig. 93, which gives two views of the same mill (the Eclipse, made by Fairbanks, Morse & Co.). In this mill there is a small rudder vane, in addition to the usual tail vane for keeping the mill to the wind, but which is normally set with its plane parallel to that of the wind wheel but projecting out at one side of the wheel as shown.

The vanes of the wind wheel in this case, as in the foregoing, are rigidly attached, and always present the same

fixed angle to the wind whether light or heavy. When the wind increases in force it acts upon the vane shown to the right of the wheel in Fig. 93, and this vane is rigidly attached to the head casting by a stout rod. The result of the unbalanced wind pressure is to turn the wheel at an angle, so that it escapes the full force of the wind. A weighted lever, connected to the rudder vane through a pair of eccentric gears (Fig. 94), acts to restore the wheel to the original position should the wind drop. This vane, the normal position of which is shown in the figure, should not be confused with the usual rudder vane to which it is set at right angles. Another expedient adopted to save the wind wheel from destruction in high winds is of a somewhat similar kind, though the unbalanced pressure necessary to turn the plane of the wheel is obtained by setting the centre of the wheel to one side of the vertical axis of rotation, the restoring couple being obtained by a spring or weight. When the wind pressure becomes too strong, it deflects the wheel, the latter is turned partially out of the wind, and thus prevents the speed of the wheel from becoming dangerously high. Another safety and governing

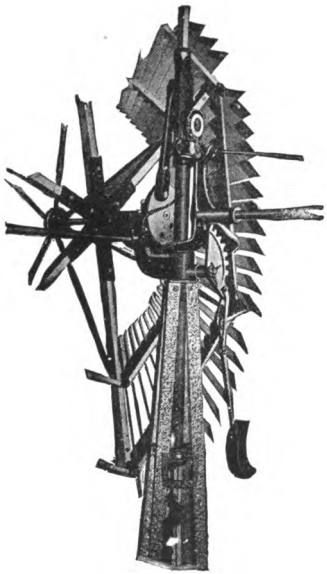


FIG. 94.—Governing Mechanism (Eclipse Mill).

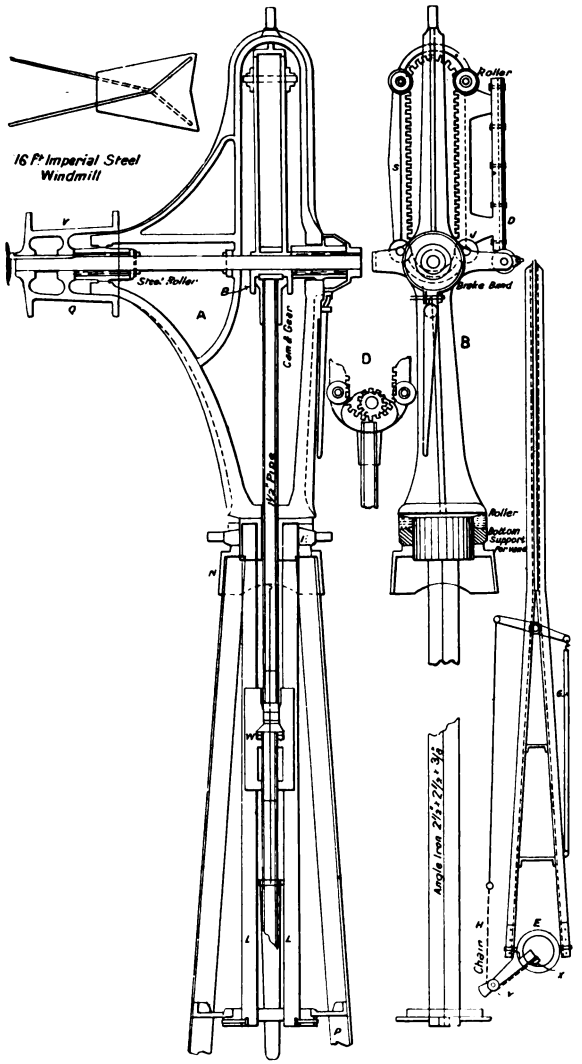


FIG. 95.—Governing Mechanism on Brantford Mill.

device, as used on the Brantford mill, is shown to the right in Fig. 95. In the normal position the wheel is out of the wind, and the wind wheel and rudder vane planes are parallel. If anything should break in a high wind the wheel goes out of the wind and consequently stops, and to start the mill it is necessary to pull the wheel round to face the wind, it being held in this position by the lanyard manipulated from the foot of the tower. The mechanism for effecting this is clearly shown in the illustration. Another mechanism by which the weathering of the sails is altered is shown in Fig. 96.

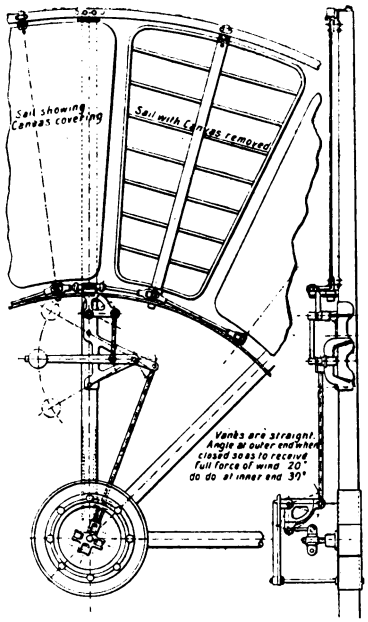

Sail showing Canvas covering
Sail with Canvas removed
Vanes are straight. Angle at outer end when closed sets to receive full force of wind 20°
do do at inner end 39°

FIG. 96.—Mechanism for Control by Weathering.

GEAR FOR STOPPING AND STARTING.

The method for stopping and starting the mill with movable sails is to set the sails at right angles to the wind, or at the angle of weather. Fig. 96 shows a wheel through which the wind may pass without communicating any turning effort to it, though the tail wheel keeps it constantly abreast to the wind. The other, and most general method of stopping and starting, is by slewing round the tail vane

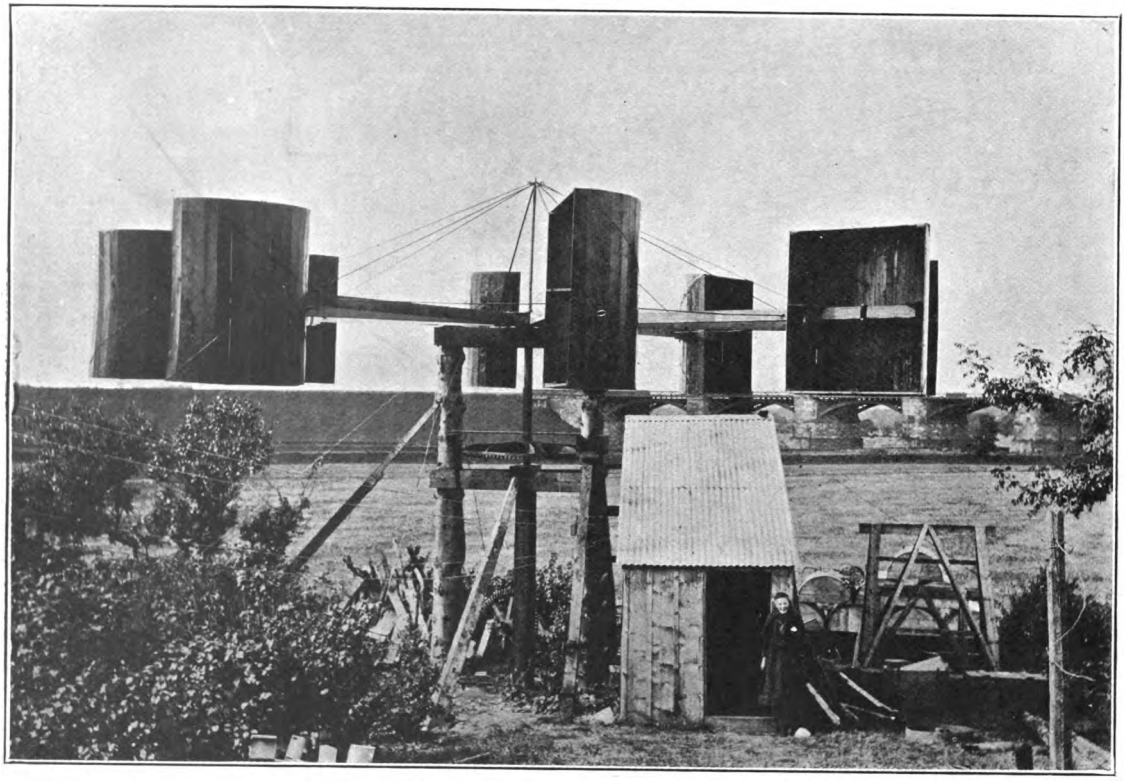


FIG. 97.—The Blyth Windmill.

through 90°, so that the planes of the wheel and vane shall be parallel. Almost all the makers of mills use this plan, the alteration in the direction of the vane being easily effected from the ground. The Brantford mill has this same method of starting and stopping, but, as we have seen, the tail vane is *normally* parallel to the wind-wheel and has to be pulled round to start the mill. Any system of brakes upon the sail-shaft is unsatisfactory, as the mill would be exposed to the full force of a heavy gale, unless the wind-wheel were turned edgewise to the wind, and kept in this position by the tail vane placed parallel to it.

The American type of steel mill being now almost the only kind made to any extent, there is little to be said about other kinds of mill. Two special designs of mill may, however, be described, as they possess novelty of design and construction.

One of these windmills, which is fashioned somewhat after the Robinson anemometer, is shown in Figs. 97 and 98 and is the invention of the late Professor Blyth. The writer is indebted to his son, Mr. V. J. Blyth, for the use of the photographs from which the illustrations have been made. Several of these mills have been erected, one at Marykirk and another at the Montrose Asylum. This latter mill suffered a fracture in the main 4-in. vertical driving-shaft and was not re-erected.

The reasons which led the inventor to adopt this type of mill are stated in a paper presented to the Royal Scottish Society of Arts.[1] He states that he discarded the old type of mill because, when it was necessary to reef the sails, the mill had to be stopped, and just at the time when it should

[1] Transactions, Vol. xiii., part 2.

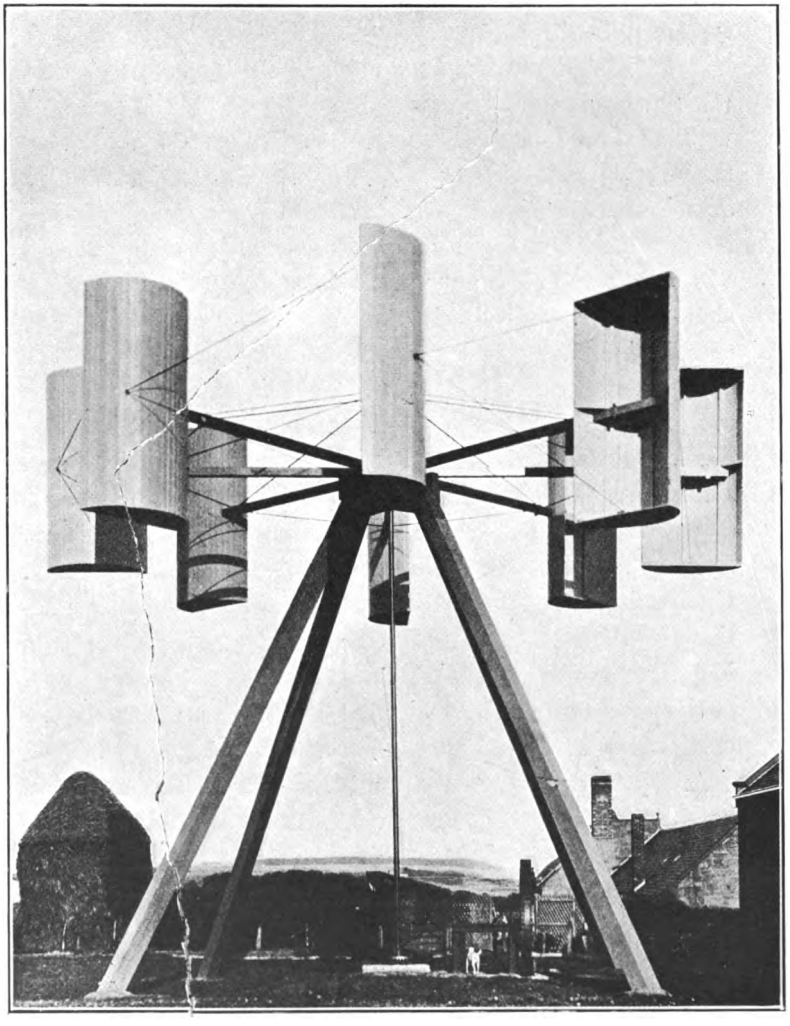


Fig. 98.—The Blyth Windmill.

have been going at its best. He then tried the American type of mill with metal sails, and, finally, in order to satisfy the following conditions, the anemometer type was chosen. (1) The mill must be always ready to go; (2) it must go without attendance for lengthened periods; (3) it must go through the wildest gale and be able to take full advantage of it. The machine he describes had four semi-cylindrical boxes attached to four strong arms, each about 26 ft. long. The opening of each box was 10 ft. by 6 ft., and the vertical shaft was an iron rod 5 ins. in diameter. At the lower end of the shaft the power was taken off by gearing, and the dynamo was driven by a belt. The turning moment was due to the difference in pressure of the wind upon the convex and concave surfaces of the sails at the ends of the opposite sail arms. Professor Blyth stated that the speed of the sails approaches a limiting value as the wind increases in violence, so that their velocity is not proportional to the wind velocity. Upon this the regulation of the mill depends, for a destructive speed is automatically avoided. This same relationship is noticed in the anemometer at very high winds, the speed of the cups falling short of what it would be were the proportionality to prevail throughout the velocity range.

Another novel type of windmill, known as the Rollason, has been working for some time. In this mill the sails are flat surfaces like paddles, projecting from a vertical shaft, and the wind is excluded from half of the wheel by a semi-circular shield which is kept in position by the tail vane. Thus half of the wheel is exposed to the wind, which, therefore, exerts pressure only on one side of the shaft and causes the wheel to revolve.

CHAPTER XII.

THE POWER OF MODERN WINDMILLS.

THE first point that an engineer requires reliable information upon in contemplating the use of wind-power is an estimate of the power it is possible to obtain from a mill, and the cost of the mill and the upkeep of the machinery. The size of the wheel alone is not sufficient to judge the power of the mill from, for an inspection of manufacturers catalogues illustrates the difference that may exist in the power of different sized wheels actuated by a wind of a given velocity. Nor can the power of a wind-wheel be ascertained by calculation from the elements of the wheel, such as sail area and angle of weather, for even if it were possible to obtain the power put into the wind-shaft, that lost in the friction of the gearing has to be taken into account, if the net power applied to the pumps is to be accurately determined.

In the previous chapter allusion was made to the available energy in a wind of given velocity, and how an estimate of the efficiency of a wheel might be calculated. As, however, the actual question of efficiency, *i.e.*, ratio of power developed to power supplied, is of less importance than the power which a wheel of a certain size will develop, the latter is the usual form in which the value of a wind-wheel is gauged. It is therefore distinct in this respect from the usual designation for a steam engine or other form of prime mover in which the actual thermo-dynamic

or mechanical efficiency is the most desirable means of estimating the worth of the machine. The efficiency of a wind-wheel may therefore be estimated by the power of a wheel of a certain diameter in a wind of given velocity—as, for instance, a 12 ft. wheel acted upon by wind of 16 miles per hour—and in this sense it is understood, though it is always necessary to state the velocity of wind, for numerous figures are used by the makers who often state the power of the mill without mention of the wind velocity. Moreover, of two mills which are to be compared, one may yield actually a greater power than the other for a certain wind velocity, while an increase in the wind velocity of a few miles an hour will reverse the position, the other mill developing more power. In other words, a mill with fixed vanes works best at a certain speed of wind, below or above which the power may fall off relatively, though most of the accurate tests made show that through a range of wind velocities the power increases with the velocity according to a law which is not directly proportional for the best wheels.

The estimation of the actual power derived from wind-wheels rests entirely upon experimental data, of which there is not very much available. Of such as there is, some of the most interesting and accurate results so far obtained are collected together in the report upon the trials of wind-pumping engines carried out by the Royal Agricultural Society of England at Park Royal in 1903. These trials were conducted with great care and in connection with accurate anemometric observations, so that for the period during which the mills were under test accurate information concerning the wind was recorded. The wind pressure was measured by two Dines anemograph instruments placed

with their vanes about 40 ft. above the ground. As the pressure fluctuated continuously the average wind velocity for a period was obtained from the diagrams of the instrument by drawing a mean line through the trace of the stylus (see Fig. 68, p. 227). A comparison between the records obtained in this manner and those of the Robinson anemometers at Kew, not far distant, proved them to be very close, so that this method of measuring was sufficiently accurate for the purpose.

The following are the regulations under which the wind engine makers competed, and which are taken from the report of the trials:—

1. The wind engines must not exceed 4 b.h.-p. with an actual wind velocity of 10 miles per hour. They must be erected on towers so constructed that the centre of the vane is 40 ft. in height from the ground level.

2. Each wind engine must be fitted with its own pump, provided with suitable suction and delivery tanks, and connections between same. Preparation must be made on the delivery pipe to receive a valve, to be provided by the Society, loaded to a pressure of 200 ft., through which the water will pass on its way to the delivery tank.

3. The actual wind velocity will be registered by a recording Dines pressure tube fixed at a height of 40 ft.

4. The wind engines will run and be under continual observation for ten hours each day, when the wind velocity and horse-power developed by the engines will be noted.

5. Each competitor may have a representative to attend to the oiling of the engine, etc., before starting; but once it is set to work each day, any subsequent interference with

the engine will be duly noted. The engine must not be interfered with after the day's work.

6. Each competitor will use his own discretion as to the diameter of wind vanes and the speed at which the engine shall run.

7. The points to which special attention will be directed are:—

(1) Stability of tower and cost of foundations.
(2) Regulation and self-governing.
(3) Ease of erection and maintenance.
(4) Size of wind engine relative to power.
(5) Price.

8. Competitors will be required to erect the engines and provide their own foundations.

9. The wind engines will remain in position until after the conclusion of the show. The competitors will not be required to pay for the space thus occupied.

10. The trials of the wind engines entered will commence in the new permanent show yard on Monday, March 2nd, being continued, at the discretion of the judges, until April 30th, 1903.

As the utility of a farm pumping wind engine depends upon other considerations than the power that can be got out of it, these were taken into account by the judges, and they are enumerated in clause 7 of the foregoing; moreover, the efficiency of the pump was not excluded, and, therefore, a bad pump would affect the worth of the wind engine proper by affecting the entire plant. But as most wind engines of these types are employed for pumping, the pump efficiency may properly be taken into consideration, and the actual power, as measured by water lifted through a

certain fixed height (in this case 200 ft.), may be taken as a measure of the power capacity of the windmill.

Both engine and pump were expected to stand the strain of a run of two months with only the same attention that the plant would have in ordinary working, and any break-down occurring during that time necessitating the attention of the maker was regarded as sufficient justification for excluding the mill from further competition.

Seventeen makers responded to the invitation of the Society by submitting twenty-two windmills, and these were erected in the Park in suitable positions so as to avoid blanketing each other. It would take us too far to describe all the details of the mills, which were the product of the leading manufacturers of the world. Besides the usual vertical wheel with tail vane, there was one in which the wheel was mounted upon the top of the tower like a mushroom on its stalk, which position it occupied when at rest or in a high wind. When working normally the axis was almost horizontal, but the action of a high wind would be to blow it up until it was edgewise to the wind.

The laurel of the competition was awarded to the mill that contained the chief points of excellence, and it is significant that the wheel that showed the highest efficiency by pumping the most water was also the one that in general excellence of design, governing and other qualities, proved superior. The points specially worthy of commendation in this mill (the Brantford) were:—

1. The general excellence of design, especially as regards the engine and pump.

2. The efficiency as determined by the amount of water pumped.

3. The successful governing.

4. The arrangement for the automatic application of the brake.

5. Economy in upkeep, due to the slow motion of its moving parts, and good workmanship.

6. Reasonable price.

The readings of the instruments were taken at frequent intervals during the trials, and the performance upon which the awards were based covered the full period under which the wheels were tested. Taking one of the readings at random we find that in a wind of 12 miles per hour the mill was pumping water against 200 ft., and by taking the amount of water pumped per unit of time the horse-power works out at 0·57. The efficiency of this wheel on the basis before outlined would be calculated thus :— The weight of air passing through a circular ring of 16 ft. diameter per second at a velocity of 12 miles per hour would be 283 lbs., and the energy expended per second in bringing this air to rest would be 1,369 ft. lbs. = 2·49 h.-p. The efficiency would, therefore, be 23 per cent., which is a high value.

The general results of the trials are chiefly of interest as a basis of comparison between mills of different makers, but they are not complete in the sense in which an engine and boiler trial would be regarded as complete. The amount of water pumped during a stated period with an average wind velocity was the principal record obtained. As the water pumped is in direct proportion to the number of strokes of the pump, it forms a rough guide to the speed of the mill in a wind of known velocity, and by studying the way in which the water pumped during the period

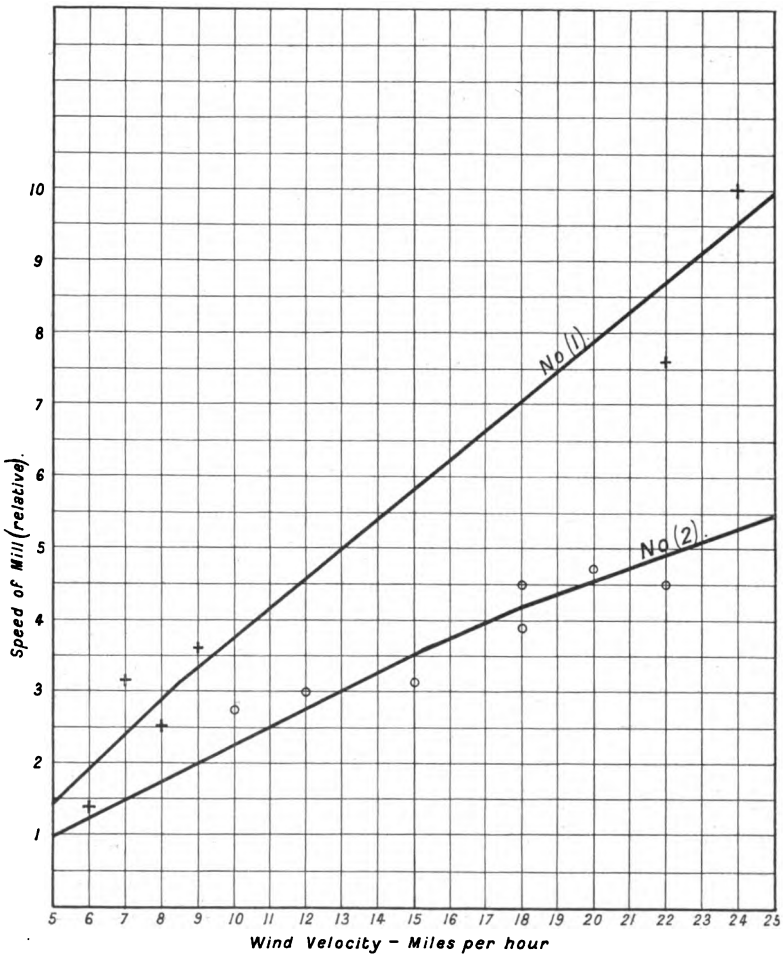


FIG. 99.—Relation between Wind Velocity and Speed of Wheel.

varies with the wind velocity the law connecting the speed of wind and speed of wheel may be exposed. Taking two

mills, referred to as (1) and (2) in Fig. 99, the traces show the variation in the amount of water pumped, and consequently the speed of the wheel with a varying wind velocity. The figures representing the speed of the mill are purely arbitrary, and the curve is therefore solely intended to point out the actual variation in wheel-speed with wind velocity. It will be seen that for both mills the trace, which is drawn through the points of observation and represents a mean value, is approximately straight, showing that the linear velocity of the sails is roughly proportional to the velocity of the wind. The curve No. 2 for a 30 ft. wheel dips down slightly at the upper end, indicating that at the higher velocities the speed of the wheel falls relatively to a slight extent. The same relationship for a 30 ft. mill is shown in Fig. 100, the curve representing a mean value of the observations. For low velocities the speed of the mill increases more than proportionally to the speed of the wind, but above 10 miles an hour the proportional law prevails. For the greater part of the range the wind velocity and wheel speed are directly proportional, as the line is straight for the most part.

The design of each tower was carefully examined, and during the trial their stability was put to a severe test, for a wind of 45 miles per hour was recorded. This would correspond to a direct pressure of about 6·1 lbs. per square foot, which, acting upon a sail area of 150 sq. ft. at the top of a 40 ft. tower, represents an overturning moment of about 36,600 lb. ft., which is resisted by the weight of tower and the tension in the foundation bolts on the windward side of the base, while the tendency to distort the tower is resisted by special diagonal wind bracing, which is shown in the

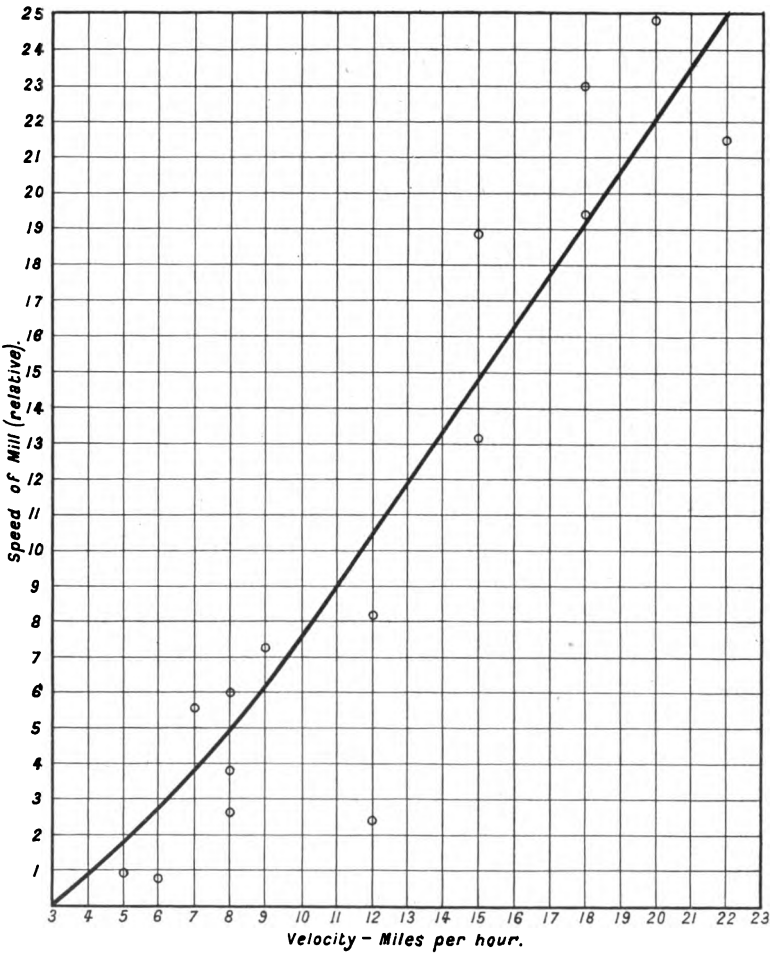


FIG. 100.—Relation between Wind Velocity and Speed of Wheel (30 ft. wheel).

illustration on p. 262. The pump attached to the Brantford mill is shown in Fig. 101, which is designed especially

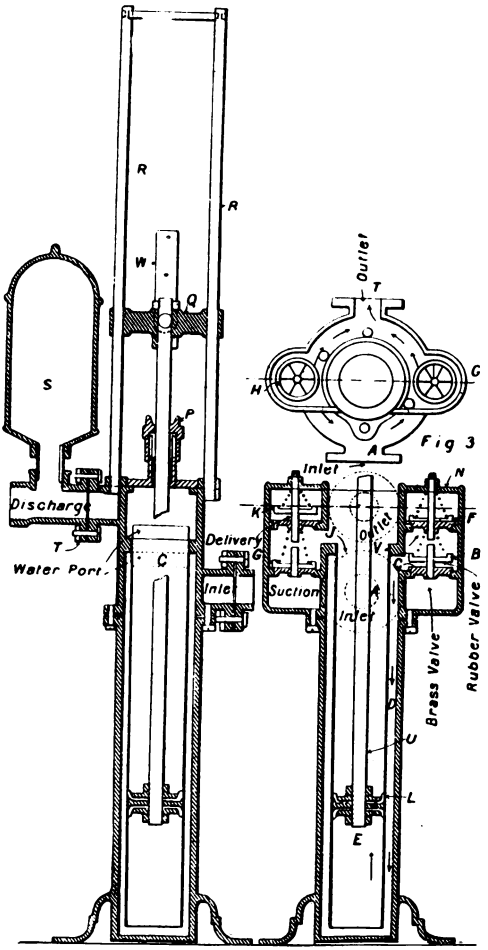


FIG. 101.—Plunger Pump for use with Windmill.

for connection to wind engines. The stroke of this pump is exceptionally long, owing to the absence of a crank and connecting rod mechanism at the mill head in

this engine. The plunger works in a gun-metal tube within the main casting, the inlet and outlet valves being placed at the top of the pump. The action is clearly illustrated in the figure, and it will be noticed that there is no valve in the plunger, both inlet and outlet valves being contained in seatings in the casting. The air vessel upon the delivery system prevents any undue shocks being thrust upon the pump in the event of a stoppage in the pipe. The pump-rod end is fastened to a cross-head which works on guide rods, and the plunger makes one stroke to two and a half revolutions of the wind wheel, and it has a diameter of 4 ins. with a stroke of 22 ins.

An important series of experiments carried out on windmills in the years 1882 and 1883 for the United States Wind Engine and Pump Company by Mr. T. O. Perry, is now published by the Department of the Interior of the United States Government, and the writer is indebted to the Director of the Department, Mr. Geo. Otis Smith, for permission to make use of the information contained therein.[1]

The results of these experiments, conducted as they were for a private company, were not made known for some years, until they were eventually printed as one of the Government papers, but they have had an influence already upon the design of windmills in view of the important points elucidated. The demand for windmills in the arid and semi-arid regions of the West for raising water which could generally be found below the surface, led to a demand for improvement in construction, so that at all times and in all

[1] Water Supply and Irrigation Papers of the United States Geological Survey.

weathers the wind engine might be safely relied upon to supply the water necessary for the cultivation of the land.

At the time these experiments were carried out the wind wheels were nearly all made with narrow wooden slats for sails, set at angles with the plane of the wheel ranging from 35 deg. to 45 deg. The slats were usually placed as close together as possible without their projections on the plane of the wheel overlapping, and the proportions of sail surface and the angles of weather were very different, as there was no apparent rule by which they were proportioned. It was with a view towards learning something on these points that these dynamometric experiments were carried out. The angles of weather and other constants given by Smeaton more than a century ago were the chief source of reference in Europe, but American makers had departed considerably from the *dicta* of this eminent engineer. The universal practice in America was to construct the wheel with slats, so that the total sail surface exceeded the total area of the annular zone containing them by more than one-fifth of the whole zone. This was in direct violation of the principles of Smeaton, and these experiments showed that it pointed in the wrong direction, and that Smeaton's results could be copied as regards the sail area with beneficial results, though the angle of weather might profitably be different.

The experiments were conducted in a room 36 ft. by 48 ft. and 19 ft. high from the floor to the roof trusses, and Fig. 102 shows a plan and elevation of the apparatus employed. It consisted essentially of a revolving arm AB centered at A and counterbalanced by a weight on the opposite side which is not shown in the illustrations. This arm could be rotated by means of the gearing and

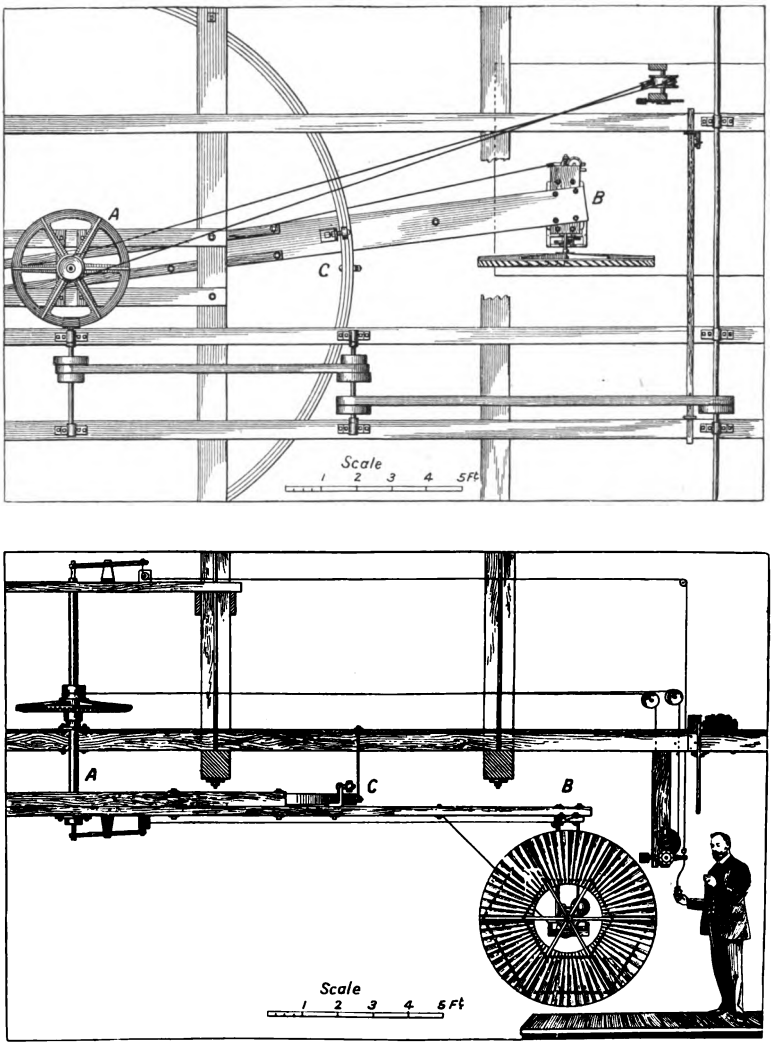


FIG. 102.—Whirling Table for Testing Windmills (Plan and Elevation).

x 2

belts at any desired uniform speed within the limits of the experiments. Two rollers C on opposite sides of the pivot A revolved upon a circular track so as to take the weight of the apparatus and prevent vertical oscillation. The wind wheel to be tested was supported at the outer end of the arm, and was thus exposed to a wind of definite velocity according to the angular velocity of the arm. The distance from the axis of the sweep to the centre of the wind wheel was 14 ft., so that the velocity in miles per hour would be given very closely by the number of revolutions per minute made by the sweep. As the air in the room was assumed to be quiescent, the actual wind velocity acting on the sails was that due to the speed of the sweeps; for though the action of the sweep carried the air with it to some extent, it did not produce a noticeable current. The sweeps were caused to revolve by an 80 h.-p. engine. The revolutions were counted by an 80-toothed wheel, so that the fraction could easily be determined to within 0·0125 of a turn. For measuring the power a prony brake was used which was placed upon the wind-shaft. The brake differed in some respects from the customary type (see p. 112), the usual wheel being replaced by two pine blocks clamped vertically upon a brass cylinder 5·25 ins. in diameter, and the adjustment was made by a cord passing across from one block to the other round small iron sheaves, and then by means of sheaves and levers up through the hollow shaft supporting the sweep, and finally fastened to one end of a lever shown in the elevation, by means of which the tension in the cord could be adjusted from the station of observation while the sweep was in motion. The axial thrust of the wind wheel shaft was sustained by a steel point.

The number of turns made by the wind wheel was observed on a worm and wheel gearing, the worm being placed upon the wind wheel shaft. Each turn of the wind wheel would cause the toothed wheel to make 0·0125 of a revolution as there were eighty teeth in the wheel, and the fractional turns could thus be estimated to within one-tenth of a revolution of the wind wheel.

In making a test the dynamometer was loaded with a weight, say 1 lb., and both the counter for recording the speed of the wind wheel and that for wind velocity were set at zero. The sweep having been set in motion, the speed of the wind wheel was checked by the brake, which could be adjusted by regulating the tension in the cord. As soon as the friction[1] and weight were balanced, both counters were thrown in by a single movement of a lever, and, at the end, both simultaneously thrown out. The friction in the journals, expressed in pounds at 1 ft. radius was added to the load. This made the total load, which was then multiplied by the turns of the wind wheel per unit of time to obtain the power. A second determination was then obtained in the same manner with a greater load applied, and then another, until the maximum was reached, for, with the diminishing speed of the wheel, there would be a value at which the product would begin to decline. The greatest product corresponded to the best load for the wheel, and the speed of the wheel at that load would be the best speed.

This greatest product would therefore be the greatest

[1] A friction test made on the journals showed that 0·27 lbs. acting at 1 ft. radius was sufficient to accelerate the wheel when once started, but it was not sufficient to overcome the statical friction.

power possible from the wheel when driven by a wind of given velocity, and would represent the maximum efficiency of the wheel.

Some particulars of the wheels tested are set forth in the following table :—

No. of Wheel.	Angle of Weather.	Sail Area. Sq. Ft.	*A.* Revolutions Per Minute at Maximum.	*B.* Revolutions Per Minute, Unloaded.	$\frac{A}{B}$
1	35	9·19	31·69	59·08	0·54
2	35	18·38	33·43	57·95	0·58
3	24·5, 11·25	13·59	47·48	90·48	0·52
4	20, 30	,,	55·93	89·43	0·63
5	22·5, 32·5	,,	46·10	86·78	0·53
6	30	10·69	42·10	69·03	0·61
7	25	,,	47·40	73·60	0·64
8	35	,,	36·84	62·67	0·59
9	32·5	,,	34·58	66·40	0·52
10	27·5	,,	39·33	72·23	0·54
11	20	,,	45·46	77·60	0·59
12	15	,,	43·88	75·60	0·58
13	40	,,	28·70	56·33	0·51
14	45	,,	24·72	48·60	0·51
15	47·5	,,	25·30	46·84	0·54
16	50	,,	20·67	42·40	0·49
17	25, 35	,,	38·70	73·90	0·52
18	25	12·94	39·42	75·58	0·52
19	27·5	,,	38·20	72·26	0·53
20	30	,,	39·87	70·78	0·56
21	32·5	,,	38·72	67·97	0·57
22	35	,,	31·96	64·20	0·50
23	25, 30	,,	43·21	73·46	0·59
24	27·5	15·00	38·87	72·60	0·54
25	30	,,	38·95	69·00	0·56
26	25	,,	43·74	75·38	0·58
27	35	,,	33·74	63·65	0·53
28	32·5	,,	34·49	67·07	0·51
29	27·5	17·02	39·10	73·10	0·53
30	30	,,	35·15	69·18	0·57
31	32·5	,,	34·94	66·73	0·52
32	35	,,	33·43	62·90	0·53
33	25	,,	37·80	75·46	0·50
34	25, 35	12·94	43·77	76·47	0·57
35	27·5	,,	46·64	76·78	0·61

No. of Wheel.	Angle of Weather.	Sail Area. Sq. Ft.	*A.* Revolutions Per Minute at Maximum.	*B.* Revolutions Per Minute, Unloaded.	$\frac{A}{B}$
36	25	12·94	47·05	80·67	0·58
37	30	,,	40·41	73·12	0·55
38	22·5	,,	52·51	84·65	0·62
39	27·5	13·07	35·57	64·52	0·55
40	27·5	12·94	45·74	82·13	0·56
41	25	,,	46·81	84·70	0·55
42	30	,,	43·15	77·95	0·55
43	25, 30	,,	45·92	84·92	0·54
44	25, 30	13·59	44·35	84·88	0·52
45	20, 30	14·16	79·99	156·63	0·51
46	17·5, 27·5	,,	85·19	160·90	0·52
47	22·5, 32·5	,,	73·31	148·68	0·49
48	25, 35	,,	66·79	141·97	0·47
49	27·5, 37·5	,,	58·82	136·30	0·43

The fourth column of the table gives the revolutions per minute that the wheel made when it was putting the maximum power into the shaft, *i.e.*, when the product of the revolutions and load was a maximum. The next column gives the speed with the wheel unloaded, and the succeeding column the ratios of these two speeds, which are, with few exceptions, about one to two. This result is largely independent of the angle of weather, though if any inferences are to be drawn it would appear that by decreasing the angle of weather the loaded speed rises so that the ratio is larger. Thus wheel No. 35 had an angle of 27·5 deg. and a speed ratio of 0·61, while No. 14 had an angle of 45 deg. and a ratio of only 0·51. It may, however, with a fair degree of truth, be said that the loaded speed is about one-half that of the unloaded speed. Of course the friction is necessarily included in the unloaded speed which cannot be got rid of, though in these

wheels it was reduced to a very low value by wet and of journals.

We have seen that the energy stored in a mass of air M moving with a velocity V is proportional to M and to V^2. But as M, the mass of air passing a given point per second, is also proportional to V, it follows that the power expended in bringing an air current to rest is proportional to V^3. Consequently, if a wheel have a fairly constant efficiency over a range of wind velocities, the power indicated by the dynamometer ought to vary as the cube of the velocity of the wind. To ascertain how nearly theory and practice conform in this respect, the experiment was tried by varying the wind velocities and recording the power corresponding to them. The power of the wheels taken was the maximum possible for each wind velocity, and it was obtained, as in the previous experiments, by varying the load on the dynamometer until the product of velocity and load was a maximum.

In the adjoined table, the four wheels, which formed the subject of these experiments, are those designated by the numbers 2, 40, 41 and 42, in the foregoing table. Each wheel was first tried at a wind velocity (a) and then the speed of the sweep was increased to (b), the maximum products (power) being recorded (A and B) under each speed. Taking wheel No. 2 under a wind velocity of 6·73 miles per hour we find that the maximum power is represented by the arbitrary figure 26·290. By increasing the wind to 8·46 miles, the power ought theoretically to be—

$$26{\cdot}29 \times \frac{(8{\cdot}46)^3}{(6{\cdot}37)^3} = 61{\cdot}6.$$

The actual figure was 62·930, from which it is seen that

Angle of Weather.	No. of Wheel.	Wind per Hour.		Product at Maximum.		$\frac{b}{a}$	$\left(\frac{b}{a}\right)^3$	$\frac{B}{A}$	$\frac{B}{A}-\left(\frac{b}{a}\right)^3$	$A\times\left(\frac{b}{a}\right)^3$
		a	b	A	B					
°		*Miles.*	*Miles.*							
	2	6·37	8·46	26·290	62·928	1·328 −	2·342	2·394	+0·052	61·571
35	„	8·46	11·04	62·928	135·498	1·305 +	2·223	2·153	− ·070	139·889
	„	6·37	11·04	26·290	135·498	1·733	5·199	5·154	− ·045	136·682
	40	6·37	8·46	36·190	86·906	1·328 −	2·342	2·401	+ ·059	84·757
27·5	„	8·46	11·04	86·906	185·856	1·305 +	2·223	2·139	− ·084	193·192
	„	6·37	11·04	36·190	185·856	1·733	5·199	5·135	− ·064	188·152
	41	6·46	8·50	38·577	88·939	1·315 +	2·274 +	2·306	+ ·032	87·724
25	„	8·50	11·02	88·939	182·985	1·297 +	2·182	2·057 +	− ·125	194·065
	„	6·46	11·02	38·577	182·985	1·706 +	4 965 +	4·743	− ·222	191·535
	42	6·43	8·46	35·580	81·985	1·317 +	2·284 +	2·304 +	+ ·020	81·265
30	„	8·46	11·0	81·985	187·294	1·299 +	2·192	2·284 +	+ ·092	179·711
	„	6·43	11·0	35·580	187·294	1·710 +	5·000	5·264	+ ·290	176·975

there is a close agreement between theory and practice. The differences in the tenth column, or $\frac{B}{A} - \left(\frac{b}{a}\right)^3$, are the discrepancies between the two, while the last column is the value of the power at the increased speed calculated from the lower speed according to the cube law. As the pressure upon the sails would theoretically be proportional to the square of the wind velocity, the starting forces ought therefore to vary as the square of the recorded wind velocity, and the following table shows the result of experiment in this direction. Two velocities of wind, a and b, give starting forces A and B, and on this assumption $\frac{B}{A}$ ought

RELATION OF DIFFERENT VELOCITIES OF WIND TO STARTING FORCES.

No. of Wheel.	Angle of Weather.	Wind per Hour.		Starting Forces.		$\frac{b}{a}$	$\left(\frac{b}{a}\right)^2$	$\frac{B}{A}$
		a	b	A	B			
		Miles.	*Miles.*	*Pounds*	*Pounds.*			
2	35	6·38	8·40	2·3	4·1	1·319	1·740	1·783
,,		8·40	11·00	4·1	7·0	1·307	1·708	1·704
,,		6·38	11·00	2·3	7·0	1·722	2·965	3·043
40	27·5	6·40	8·52	2·0	3·65	1·330	1·769	1·825
,,		8·52	10·94	3·65	6·2	1·285	1·651	1·699
,,		6·40	10·94	2·0	6·2	1·709	2·921	3·100
41	25	6·40	8·50	1·9	3·35	1·327	1·761	1·763
,,		8·50	11·11	3·35	5·5	1·308	1·711	1·642
,,		6·40	11·11	1·9	5·5	1·735	3·010	2·895
42	30	6·41	8·47	2·2	3·8	1·322	1·748	1·727
,,		8·47	11·06	3·8	6·8	1·305	1·703	1·789
,,		6·41	11·06	2·2	6·8	1·725	2·976	3·091

to be equal to $\left(\frac{b}{a}\right)^2$. The last two columns show the differences between the two values which, considering the

inevitable inaccuracies of experimental research, are sufficiently close to attest the truth of the rule that the pressure varies as the square of the velocity. The measurement of the starting forces is liable to inaccuracy; moreover, as the author of the experiments states, they could not be defined or determined with the same accuracy as the maximum product of speed and load. It would seem that the measurement of a starting force would in itself be especially difficult, and the only absolutely correct method of determining it exactly would be to measure the angular acceleration produced by the wind pressure. By exposing the wheel to a wind with a load, at first sufficient to prevent turning, and gradually decreasing the load until acceleration begins, a rough measure, depending upon visual observation, is obtained. An important result of these experiments upon wind wheels was the effect produced by cutting out obstructions from between the sails which produced aerial resistance to motion.

The conclusions arrived at regarding the sail area and best number of sails were, that wide sails do better than narrow ones, and that it is better to divide a given sail surface between a few sails than a large number. Thus one wheel with only six sails gave 2·5 times the efficiency of another with sixty sails. The reduction of the number of sails reduces the aerial resistance, and also leaves relatively fewer interstices for the air to flow through. Taking the total area of the zone formed by the space swept through by the sails as 100, three wheels were tested having respectively 75, 87·5, and 100 per cent. of sail area. Each wheel had the same number of sails differing only in width, and the maximum products are represented by the

relative figures 1·204, 1·212, and 1·201. It is evident therefore that nothing was gained by making the total sail area more than 87·5 per cent. of the zone area, and that 75 per cent. was nearer the maximum than 100 per cent. The actual horse-power of these wind wheels of 5 ft. diameter is very small. Taking the result of one of the trials, we find that under a wind of 8·40 miles per hour the load (including 0·1 lb. for friction) at which the wheel gave the maximum power was 1·9 lb. applied at a distance of 1 ft. from the axis of the wind-shaft, and the wheel was making 32·9 revolutions per minute with this load. The energy absorbed by the brake per minute would therefore be $1{\cdot}9 \times 2 \times 3{\cdot}14 \times 32{\cdot}9 = 393$ ft.-lbs. or about 1·2 per cent. of one horse-power. By taking the energy in the wind (see p. 274) and that spent at the brake the efficiencies of these wheels were determined. The highest efficiency on this basis for any of the wheels tested was 0·29. The maximum power obtained from any of the 5-ft. wheels was developed in a wind of 10·9 miles an hour, and was 1,553 ft.-lbs. per minute, or about one-twentieth of a horse-power. To obtain one horse-power from this wheel the velocity of the wind in miles per hour would therefore have to be

$$\sqrt[3]{\frac{33{,}000}{1553}} \times 10{\cdot}9 = 30{\cdot}2.$$

As the areas of circles are to each other as the squares of their diameters, the sail area of a wind wheel likewise increases with the square of the diameter. But by increasing the size of the wheel its weight must be enormously increased to provide strength to withstand the increased wind pressure. If all the linear dimensions of a wheel were

doubled its weight would be eight times the original, but at the same time the sail surface would only be four times as great. Consequently, there is a limit to the size, above which the wheel becomes so heavy that for light winds it is useless. For this reason small wheels are the best, as, without undue weight, they can be made sufficiently strong to withstand heavy wind pressure, and the cost of construction per unit of power is not any larger in small wheels than in large. This is at variance with other forms of motor to some extent, in which the cost per horse-power is greater for the smaller than the larger sizes. Mr T. O. Perry, in referring to an experiment he made on a 22-ft. wind wheel in natural wind, states that, though the anemometer was placed as close to the wind wheel as possible, it was observed that the instrument would sometimes stop running when the wind wheel was actually accelerating, and that the wind could often be heard whistling through the sails on the opposite side of the wheel, while very little wind was felt on the near side close to the anemometer. In regard to the multiplication of wind power, he writes: "That for equal safety in storms, the weights of wind wheels of different sizes and like forms should be proportioned to the cubes of their diameters. It would require four 12-ft. wheels to equal the area and power of one 24-ft. wheel if the larger wheel is proportionately elevated. But the weight of the one 24-ft. wheel would be twice as great as the combined weight of the four 12-ft. wheels, and the weight of the one higher tower would probably be twice that of the four shorter towers combined. Hence it would seem that in proportion to the power obtained in each case, the one 24-ft. wheel would cost

twice as much in material. The thought naturally presents itself that the four 12-ft. wheels ought in some way to be combined so as to act in unison for concentrating a great amount of power where it is desirable to use the power at only one point, as in driving one machine of large dimensions. If the four wheels were coupled together rigidly, the trouble from uneven reception of wind which is experienced in large wheels would be augmented. The problem has not been worked out, but we may imagine a number of wind wheels, each compressing air according to its own ability and delivering it at any distance into a common reservoir. Natural elevations would be selected as locations for windmills, and such a plant could not be rendered useless for the time by an accident to one or two of the wind wheels. There would necessarily be considerable loss in compressing air, but a low pressure system might be devised that would greatly reduce the waste. Some waste of power attends every mode of transmission. In seeking to make a gain in power of 100 per cent. in proportion to cost of plant, the loss of an extra 25 per cent. in transmission might well be tolerated. There may, however, be other and better methods for accomplishing the object in view than by the means we have ventured to suggest."

Assuming the cube law to hold, the power that may be derived from a wheel of the best construction at different wind velocities would be approximately as in the following table, though a word of caution is necessary lest the figures may be taken as expressing more than the available evidence on the subject justifies. The great variety of conditions and dimensions of wheels of different makers necessarily renders such an estimate only approximately

reliable, but as a guide to an intending purchaser of a wind engine the figures may have some measure of significance :—

HORSE-POWER OF A 16-FT. WIND ENGINE AT VARIOUS WIND VELOCITIES.

Wind Velocity. Miles per Hour.	Horse-Power developed by 16-ft. Wheel.
4	0·02
6	0·06
8	0·15
10	0·30
12	0·50
15	1·00
20	2·3
25	4·5
30	7·8
35	12
40	18

To derive an estimate of the power that may be expected from a wheel of different diameter than 16 ft., multiply the horse-power as given in the table by the ratio of the squares of the wheel diameters. For instance, the power of a 12-ft. wheel in a 15-mile wind would be approximately

$$1{\cdot}00 \times \frac{(12)^2}{(16)^2} = 0{\cdot}56 \text{ h.-p.}$$

The increasing friction losses and aerial resistance, besides the wide difference in linear velocities of the outside and inside of the sails on large wheels, render these estimates for the power less accurate for wheels above 20 ft. in diameter, and, as far as experiments go to show, the power

does not increase according to the same laws above that size. They are, therefore, applicable to the most generally used size of wheels, which range from 10 to 16 ft. in diameter. For a 10-mile wind the h.-p. of wheels from 12 ft. to 30 ft. in diameter would be approximately as follows :—

Diameter of Wheel.	Horse-Power.
12	0·17
16	0·30
18	0·38
20	0·47
25	0·74
30	1·05

The following figures by Mr. Murphy are the result of laboratory tests on windmills at wind velocities ranging from 10 to 25 miles per hour :—

Size of Mill.	Wind Velocity.		
	10 Miles per Hour.	15 Miles per Hour.	20 Miles per Hour.
12 ft.	0·21 h.-p.	0·58 h.-p.	1·05 h.-p.
16 ft.	0·29 ,,	0·82 ,,	1·55 ,,

For higher wind velocities it is found that the values fall much under the theoretical values.

Mr. A. M. Orr records some careful prony-brake tests which show that in a 25-mile wind an 8-ft. wheel gave

0·12 h.-p. in pumping water, and a 12-ft. wheel developed 0·64 h.-p. The following are the results obtained from a 30-ft. wheel which was employed to lift water into a tank through a distance of 135 ft.

Velocity of Wind. Miles.	Cubic feet of Water pumped per minute.	Horse-Power.
8	0·89	0·23
12	2·25	0·58
16	3·00	0·77
20	3·60	0·92
25	4·20	1·07

The actual power at the wind-shaft was of course greater than these figures, as the friction in the pipe line and pump mechanism absorbs a large percentage of the power developed by the mill.

Windmills Applied to Electric Installations.

It might be supposed that cheap electricity would be possible with the windmill at our disposal to drive our dynamos, and that the acquisition of "power for nothing" would solve the problem of electric lighting, and banish the small isolated steam plant and internal combustion engine. But such is by no means the case, for, with few exceptions, and these expensive ones, the windmill electric installation has proved to be a failure so far, and has no compensating advantages for the doubtful virtue of obtaining power from the winds that blow, and thus avoiding a fuel bill. As dynamo driving requires a constant speed if the lamps on the circuit are directly connected to the machine, and are

to work at constant potential, it is clear that the machine cannot be directly connected to the windmill unless some sort of governing mechanism be provided whereby the fluctuation in speed shall be compensated by the field strength of the machine. This expedient has been tried, but without success, and we are therefore thrown back upon the storage battery as the only way out of the difficulty. With the introduction of the storage battery great complication ensues, besides the addition of an appliance which requires unceasing attention for its proper working. By such arrangements the dynamo supplies current to the cells only when the speed is up to an assigned amount, and the current is drawn off for the lamps at the proper voltage.

The writer has received several letters from men who have tried installations of this character, all of which tell the same story. The irregular speed of the mill damages the accumulators rapidly, and as there is no other means of utilising the variable energy, they have been taken out in most cases. Devices for storing the mechanical energy have been tried, one of which consisted of a system of weights which could be raised by the windmill and which drove dynamos through a system of rope gearing during their descent in the manner of a clock weight. The enormous size of the weights necessary and the great friction-losses rendered this method unpractical. Supposing a drop of 60 ft. were possible it would necessitate a weight of 14·7 tons falling through that distance in an hour to give one horse-power, friction excluded. The loss in friction would probably require double this weight to obtain one horse-power at the dynamo shaft. Two such weights, with a very heavy structure to support them, would at least be

necessary, so that when one was being raised by the mill the other would be giving out energy by falling. Such a plant would not be free from stoppage owing to low winds, and to increase the number of weights, so as to have one horse-power available at any time, would necessitate such an outlay as to be altogether prohibitive, even if absolute reliability were assured.

The method of storage by raising water to be afterwards used in a turbine driving a dynamo is also quite prohibitive in cost. Supposing, for instance, sufficient water has to be stored so that it would yield one horse-power for 20 hours, and that the tank is placed 70 ft. above the turbine. The capacity of such a tank would have to be about 9,000 cubic ft., and its dimensions 24 ft. in diameter and 20 ft. deep. It is probable that scarcely more than half a horse-power would be obtained as electrical energy in view of the very low efficiency of a turbine and dynamo of such a small size, so that to obtain one electrical horse-power the tank would require to have double this capacity.

The available power being exceedingly small, and a large proportion of it being lost in the gearing and generator, the actual energy put into a dynamo is very small for the size and cost of the plant. Taking a 12-ft. windmill giving 0·17 h.-p. in a 10-mile wind, and driving a dynamo of 80 per cent. efficiency, the electrical output would be $0{\cdot}17 \times 0{\cdot}8 = 0{\cdot}136$ h.-p. $= 0{\cdot}101$ kw. This would be sufficient for a photometric power of $101/3{\cdot}5 = 29$ candle power constantly burning, assuming 3·5 watts per c.-p., or almost two 16 c.-p. incandescent lamps. It has been found that a 12-ft. wheel would provide a minimum of 19 lamp-hours (16 c.-p.) per day, while a 16-ft. wheel would give 35 lamp-

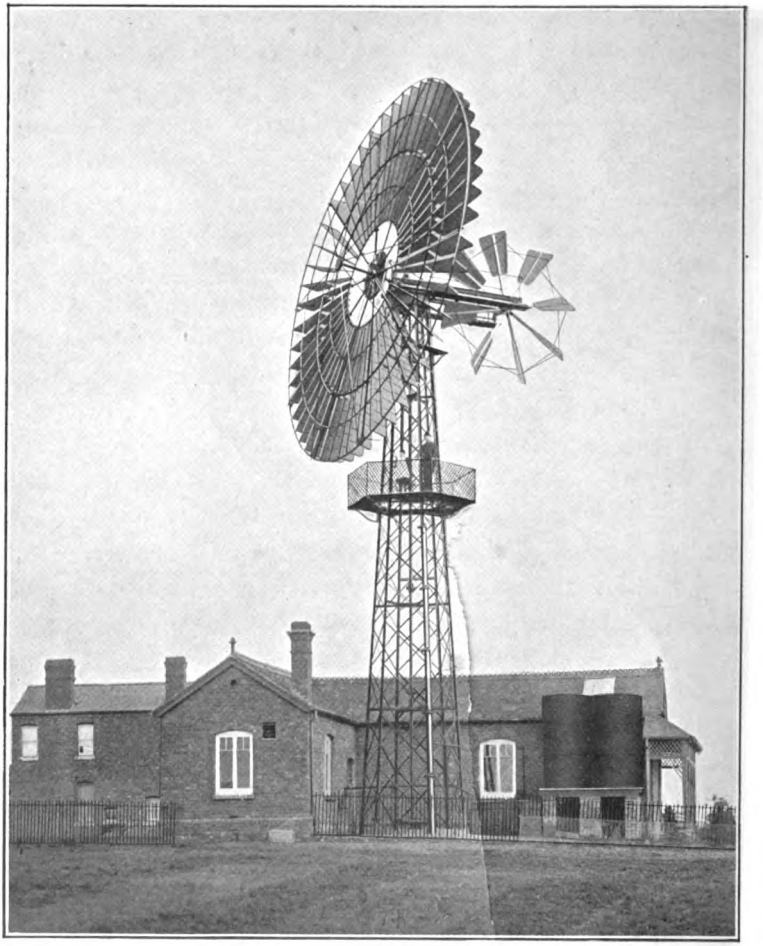


FIG. 103.—Windmill (35 ft. dia.) utilized for Pumping and Driving an Electric Generator.

hours in the same time. This was ascertained from a plant consisting of a 12-ft. wheel driving a dynamo of 0·75 kw. capacity. As the cost of such an installation, including storage batteries, would be at least £200, the economy of lighting by a wind-driven plant is very questionable. Other devices than storage batteries have been interposed to maintain a constant voltage. One of these, an example of which is at work in Indiana, is connected to a 14-ft. windmill on a 50-ft. tower. The mill drives a plunger pump, which delivers water to a reservoir in which a constant pressure of 75 lbs. per sq. in. is maintained by weights on the plunger in the reservoir. This water under pressure is used to drive a 0·5 h.-p. turbine, which is direct connected to a 0·25 h.-p. 25 volt dynamo for charging a storage battery. The battery consists of eleven cells, and lights twenty 8 c-p. lamps for three hours, or five 8 c.-p. lamps for six hours. By this means all the power of the windmill may be stored to be used in the turbine when desired. The author has been unable to obtain any information concerning the working of this installation which is of such a novel character. It is said that a windmill will run in this district for five hours a day.

The first requisite to the success of a wind-driven electric plant is a variable speed dynamo, which, within a certain speed-range, will deliver current at a constant voltage, so that accumulators, if used, may be worked to better advantage. Several protracted attempts have been made to employ wind-power for dynamos, and in several cases every possible plan has been tried to make them successful. One of these, which is shown in Fig. 103, is situated on an eminence in the private grounds of Mr. George Cadbury, and has now

been in service for thirteen years (though not all the time for electric lighting), and the canvas sails have been once replaced during that time. The canvas is treated with lead paint, which preserves the material very well. The motion of the sails is communicated to a counter shaft, from which a dynamo is driven. Another horizontal shaft takes off power through a pinion from the same gear, by which plunger pumps are driven. A clutch enables the pumps to be thrown out of gear when it is desired to work the dynamo. The irregularity in the speed and uncertain character of the driving power has resulted in the failure of the electric plant, and a gas engine is now installed for the purpose of lighting. The accumulators were found to wear out rapidly owing to the treatment to which they were exposed, and the plates buckled. The engineer in charge of the plant informed the writer that every possible expedient had been tried to make the plant a success, but without avail. The windmill is now used for pumping, for which purpose it is admirably adapted. It is 35 ft. in diameter, and is in an exposed situation. The governing of this mill is automatic, the sails being hinged so that the angle of weather is varied, and the tendency of the sails to turn edgewise to the wind is checked by a weight at the base of the tower, which hangs at one end of the furling lever, the other end being capable of being fixed into any desired position by means of a pin passing through holes in guides on each side.

The engineer in charge of this plant had an experience which few men would have gone through without injury or loss of life. One day, during a gale of wind, he ascended the tower for the purpose of removing two broken sails.

To accomplish his object he was obliged to stand on the outer circumference of the wheel, instead of on the platform. While in this position the wind carried one of the sails out of his hand, which dropped and struck the pin holding the furling lever in place. This pin was knocked out, the furling lever fell, and the sails were immediately turned for the wind to act upon, and the wheel consequently began to revolve. Hanging on with hands and feet as best he could, he was carried round several times before his shouts arrested the attention of a man who was 150 yards away and who stopped the wheel, but only just in time, for he was becoming exhausted, and his hands would have been wrenched loose had he been exposed much longer to such a severe strain.

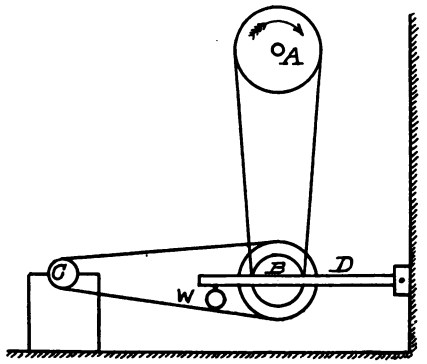


FIG. 104.—Transmission Gear between Windmill and Dynamo for obtaining constant Speed of the latter.

Among the most satisfactory experiments that have been conducted upon the generation of electricity from wind power are those of Professor La Cour, which have received the support of the Danish Government. The experimental station was erected at Askov, and after a long period of trial it has been found sufficiently satisfactory to encourage the installation of plants at other places in Denmark, more

than thirty of which are now in operation. The mill used at Askov is not built in accordance with American practice; it is rather more like the old type of mill, with four arms carrying adjustable slats crosswise which are capable of being weathered by a system of rods and levers. These four arms are 7·4 metres long, and 2·5 metres wide, giving a sail area of 74 sq. metres. It is geared to two 12 h.-p. dynamos through a vertical shaft from which the power is taken off by bevel gearing on to a horizontal shaft which enters the house. From this shaft the dynamo is driven through an arrangement of pulleys as shown in Fig. 104. The shaft A receives motion from the windmill, and by means of the two pulleys the shaft B, which is suspended in a cradle D, is turned. By altering the weight W at the free end of the cradle the tension in the belt can be adjusted. By trial it is found that when the difference between the tensions in the tight and loose sides of the belt exceeds a certain value the belt starts slipping, and consequently the dynamo, C, has a limited speed. By an ingenious switch, the possibility of the cells discharging through the battery, if the dynamo voltage were to fall below the battery voltage, is prevented. The plant is assisted by a petrol motor to cover the periods of low wind.

The Cost of Power Generated by Windmills.

The cost of wind-power is difficult to determine or to estimate with any approach to accuracy, for it depends very largely upon the point of view from which we regard it. Being a very unreliable source of power and altogether outside of immediate control, if gauged by such a criterion it

would be very costly indeed. If, on the other hand, we regard it as a means whereby water may be lifted for farm purposes, and which may therefore be independent of control, it would be regarded as a cheap and entirely satisfactory power. For most purposes where power is required reliability is the chief requisite, and it cannot be measured directly in money. For industries requiring a constant and unfailing power day by day wind wheels would necessarily be supplemented by some other form of motor, such as a gas engine, and the cost of wind-power in such a case would be great, for the very small power to be obtained from wind wheels might be developed by the gas engine with a small increase in size and, consequently, very little extra capital outlay. The combination of wind wheels with other forms of motor is unsatisfactory, and it has none of the elements of satisfactory working that are brought about by the combination of an hydraulic turbine and heat engine. The capital cost of a wind wheel per horse-power is, relatively to other forms of motor, high, and there can, therefore, be no monetary advantage except in the absence of fuel charges, which, in a combined plant, are present almost to the same extent as if the heat engine were operating alone, because of the irregular and intermittent assistance rendered by the wind engine when exposed to the variable weather in our latitude. Most industries require power constantly and in unvarying amount, and for some of them excess power cannot be used advantageously and is therefore wasted when developed. Consequently, of the total number of horse-power hours developed per annum by a wind engine, or which could be developed if it were allowed to take advantage of the gales, a large part would ordinarily be wasted, just as if a gas

engine driving a machine requiring 15 h.-p. were constantly operating at 25 h.-p., the difference being absorbed by a brake. The use of electric storage batteries prevents the waste of excess energy, but the cost of this extra plant must then be considered. The actual number of hours throughout the year at which a wind engine may therefore be relied upon to develop a full required power (which we will suppose would be developed at a wind velocity of 10 miles per hour) is disclosed by an examination of the weather charts, which shows how very different the average wind velocity is in different parts of the country. On the Atlantic coast there are a comparatively large number of hours during the year at which a velocity of 10 miles an hour or more may be counted, while at inland stations the average is very much less. Taking 3,600 10-mile hours for the year, the number of horse-power hours for a 16-ft. wheel would be 0·30 × 3,600 = 1,080 h.-p. hours for the year. The cost of a 16-ft. wind engine erected on a 40-ft. tower would be approximately £65, so that the cost of power per horse-power hour would be as follows :—

Interest @ 3½ per cent. . . .	546*d.*
Depreciation @ 7 per cent. . . .	1,092*d.*
Upkeep, including occasional attendance and lubrication, £5 . . .	1,200*d.*
	2,838*d.*
Cost per h.-p. hour	2·6*d.*

The assumptions upon which these figures rest will appear liberal when the meteorological records of English stations are examined. At most situations it cannot be

assumed that, for one half the year, there is a wind velocity of 10 miles an hour. On the other hand, as against the wind engine, it is assumed that all velocities above 10 miles an hour are grouped under that designation, so that for some part of the time the engine is developing power for which no account has been made. But, as previously explained, excess power above a predetermined amount is largely wasted, and a wind engine plant would be designed to give full load—at say 10 miles per hour—in the same way that a steam engine would be installed under a rated capacity. The cost per horse-power is therefore high for the wind engine, and there must be, therefore, some feature to counteract it to account for the rapid extension and enormous trade done in small windmills, especially for pumping water on farms in the great agricultural countries. The reason lies in the fact that, for water pumping, neither constant power nor power in a definite amount is necessary, as a pump can work under any wind down to a velocity sufficient to overcome the friction, and up to any speed which is only limited by the capacity of the machinery to escape damage. If the power developed and utilised by one of these pumping engines were integrated throughout a year it would probably be as low in cost per horse-power as in other forms of prime mover, though the distribution of the power makes it most undesirable for most purposes. Captain Scott in his notable voyage of exploration to the Antarctic regions took out a windmill on board the ss. *Discovery* for driving his electric lighting plant so as to effect a saving of valuable fuel. For such a purpose and in such a situation the windmill would appear to be ideally adapted as a supplement to other means of

lighting the ship, though in this case the design of the mill was not properly adapted to the purpose.

The initial expense of erecting a windmill is not great, as the only foundation necessary is four small concrete blocks for embedding the foundation bolts. In some mills the tower is the most expensive part of the outfit, and it is only with elaborately geared wheels that the cost of the engine exceeds that of the tower. The price of mills, erected upon towers 40 ft. high, would be approximately :—

Approximate Cost of Mill Erected Complete on 40-ft. Steel Towers.

Diameter of Wheel.	Price in £ sterling.	Horse-Power (10 Miles per Hour).
12	40	0·17
16	70	0·30
18	110	0·38
20	170	0·47
30	350	1·05

The makers frequently express the power of a mill as the number of gallons raised per hour through a height of 100 ft. To reduce this to horse power, divide the number of gallons (Brit. imp. gallon of 10 lbs.) by 1980.

The following is taken from a maker's catalogue :—The water raised through 100 ft. in gallons per hour by a 16-ft. mill under a wind of 10 miles per hour is 700. The horse-power is therefore 700/1980 = 0·35.

APPENDICES.

APPENDIX A.

To find the value of V in this equation which will render the power a maximum it is necessary to ascertain the rate of change in the power for different values of V.

$$y \text{ (power)} = \frac{A\,\gamma}{g}(v - V)^2 V.$$

$$\frac{d\,y}{d\,V} = \frac{A\,\gamma}{g}\left[(v - V)^2 - 2\,V(v - V)\right]$$

$$\frac{d\,y}{d\,V} = \frac{A\,\gamma}{g}\left[3\,V^2 - 4\,v\,V + v^2\right]$$

The power is a maximum with such a value of V as will make $\frac{d\,y}{d\,V} = o$.

$$\text{Putting } 3\,V^2 - 4\,v\,V + v^2 = o,$$

and solving for V we obtain, as the roots of this quadratic, $\frac{v}{3}$ and v. The power is therefore a maximum when $V = \frac{v}{3}$ and a minimum when $V = v$.

APPENDIX B.

Experiments made at the weir at different stages of the river showed that when the depth on the weir was 3 in. the actual fall was 6 ft.; and during an exceptionally heavy flood, when there was 18 in. of water coming over the weir, the head had dropped to 3 ft. Though the actual working head H and the weir-depth h are not connected by a linear law, the error in assuming such to be the case is small between narrow limits, so that we have

$$\mathrm{H} = b - \left(\frac{b}{a}\right)h$$

where a and b are the intercepts on the co-ordinate axes of h and H respectively. The observations supply us with a means of obtaining values to insert for a and b, for we have

$$\frac{b}{a} = \frac{6-3}{1{\cdot}5-0{\cdot}25} = 2{\cdot}4.$$

Also

$$b = 6 + (2{\cdot}4 \times 0{\cdot}25) = 6{\cdot}6;$$
$$\therefore \mathrm{H} = 6{\cdot}6 - 2{\cdot}4\,h.$$

The horse-power per foot of weir length in terms of h will therefore be

$$\text{H.-P.} = \frac{62{\cdot}5}{33{,}000} \times 200\ \sqrt{h^3}\,(6{\cdot}6 - 2{\cdot}4\,h).$$

$$(1)\quad \text{H.-P.} = 0{\cdot}379\ \sqrt{h^3}\,(6{\cdot}6 - 2{\cdot}4\,h).$$

To find the value of h, which will make the horse-power a maximum, we have

$$\frac{d\,(\text{H.-P.})}{d\,h} = 0{\cdot}379\left[9{\cdot}9\,h^{\frac{1}{2}} - 6\,h^{\frac{3}{2}}\right].$$

Putting this $= o$, we have

$$-6\,h^{\frac{3}{2}} + 9{\cdot}9\,h^{\frac{1}{2}} = o.$$
$$\therefore h = 1{\cdot}65 \text{ ft.} = 19{\cdot}8 \text{ in.}$$

We thus find that when the height of water on the weir is 19·8 in. the maximum horse-power is attained. Any further increase in h would not augment the horse-power, owing to the rapid rise of the tail-water and consequent diminution of effective head. The maximum horse-power for the full weir length under any conditions would therefore be

$$\text{H.-P.} = 167 \times 0{\cdot}379 \times 2{\cdot}12\,(6{\cdot}6 - 3{\cdot}96) = 354.$$

APPENDIX C.

If t be the thickness of a pipe and r the internal radius expressed in the same units, the stress f per unit of area acting to tear the pipe asunder, when p is the unit pressure, is expressed by the following equation.

$$f = \frac{\left(r + \frac{t}{2}\right)p}{t}.$$

In this formula the radius taken includes half the thickness of the

pipe, but this may be neglected as t is always small compared with r, therefore—

$$f = \frac{r\,p}{t} \text{ or } \frac{D}{t} = \frac{2\,f}{p} \text{ where } D \text{ is the internal diameter.}$$

The strength of the steel used in the manufacture of high pressure pipes may be taken as 60,000 lbs. per sq. in., or 4,220 kg. per sq. cm. = 42·2 kg. per sq. mm. If h is the head of water in metres acting on the pipe

$$p \text{ (kg. per sq. mm.)} = \frac{h}{1000}.$$

It is usual to assume a factor of safety of 4 so that the maximum allowable stress in the steel would be $f/4 = 42{\cdot}2/4 = 10{\cdot}55$ kg. per sq. mm.

We have therefore

$$\frac{D}{t} = \frac{2 \times 10{\cdot}55}{\frac{h}{1000}} = \frac{21{,}100}{h}.$$

If the pipe be riveted longitudinally, the resistance to rupture ought to be taken at 80 per cent. of that of the solid plate; the thickness in millimetres would therefore be—

$$t \text{ (riveted pipe)} = \frac{D\,h}{21{,}100} \times \frac{1}{0{\cdot}80} = \frac{D\,h}{16{,}880} + 3.$$

For welded pipe 95 per cent. of strength of solid plate should be taken,

$$t \text{ (welded pipe)} = \frac{D\,h}{21{,}100} \times \frac{1}{0{\cdot}95} = \frac{D\,h}{20{,}045} + 3.$$

The constant 3 is added to allow for the tendency of a large pipe to assume an oval form when lying on the ground.

Example (1). Supposing a riveted pipe of 450 mm. diameter, made of open hearth steel, be subject to a hydrostatic pressure due to a head of 800 metres. What is the thickness required with a factor of safety of four?

$$t = \frac{450 \times 800}{16{,}880} + 3 = 25 \text{ mm.} = 1 \text{ in. (approximately).}$$

Example (2). Supposing a welded pipe of 750 mm. diameter, made of open hearth steel, be subject to a hydrostatic pressure due to a

head of 600 ft. (183 m.). What is the thickness required with a factor of safety of four ?

$$t = \frac{750 \times 183}{20{,}000} + 3 = 10 \text{ mm.} = \frac{13}{32} \text{ in.}$$

If D and h are in inches and feet respectively, the two expressions become, for the thickness in inches—

$$t \text{ (riveted pipe)} = \frac{h \times D}{55{,}200} + 0{\cdot}12.$$

$$t \text{ (welded pipe)} = \frac{h \times D}{65{,}600} + 0{\cdot}12.$$

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